

Appendix 8: How to Calculate Mean, Standard Deviation, Relative Standard Deviation, and Relative Percent Error

1. Calculating the mean (\bar{x}):

All measurements have random errors (errors that are unpredictable), therefore it is more reliable to repeat a measurement several times and report the mean. Calculating the mean reduces the effect of these random errors. The mean, \bar{x} , is calculated using the equation 1.

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} \quad \text{Equation 1}$$

Where:

\bar{x} = the mean of the set of measurements

$x_1 \dots x_n$ = each value in the set of measurements

n = the total number of measurements in the set of measurements used to determine the mean; exact number (counted not measured) so it will not limit significant figures

Example 1: Calculate the mean for the following data:

Data set for time of reaction:

Trial number	T1	T2	T3	T4	T5
Time (sec.)	22.5	21.7	21.4	22.9	23.1

Answer:

$$\text{Mean} = \bar{x} = \frac{22.5 + 21.7 + 21.4 + 22.9 + 23.1}{5} = \frac{111.6}{5} = 22.32 = 22.3 \text{ sec}$$

2. Calculating standard deviation (s):

Standard deviation is a measure of how dispersed the data is in relation to the mean. In other words, it demonstrates precision by showing the variation in a set of values around the mean. The data set has a higher precision when the standard deviation has a small value. The standard deviation (s) is calculated using equation 2.

$$\text{Standard deviation} = s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}} \quad \text{Equation 2}$$

Where:

x_i = each value in the set of measurements used to determine the mean

\bar{x} = the mean of the set of measurements

Σ = the sum or addition

n = the total number of measurements in the set of measurements used to determine the mean; this is an exact number (counted not measured) so it will not limit significant figures

Standard deviation has the same unit of measure as the mean. Apply the rule of subtraction to determine how many digits to report it with.

Example 2: Calculate the standard deviation for the following data:

Data set for time of reaction:

Trial number	T1	T2	T3	T4	T5
Time (sec.)	22.5	21.7	21.4	22.9	23.1

First calculate the numerator, $\sum(x_i - \bar{x})^2$. Notice that this is the same data set used in Example 1, so we already know the mean - 22.32 sec. (When doing multistep calculations, apply rule for that step, but do not round, instead, keep a few extra digits to avoid losing precision during the remaining calculations, keep track of significant figures by underlining them. Round only the final answer.)

$$\begin{aligned} \sum(x_i - \bar{x})^2 &= (22.5 - 22.32)^2 + (21.7 - 22.32)^2 + (21.4 - 22.32)^2 + (22.9 - 22.32)^2 + (23.1 - 22.32)^2 \\ &= 0.0324 + 0.3844 + 0.8464 + 0.3364 + 0.6084 \\ &= 2.208 \end{aligned}$$

Next, substitute in the equation:

$$\begin{aligned} s &= \sqrt{\frac{\sum(x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{2.208}{5 - 1}} = \sqrt{\frac{2.208}{4}} = \sqrt{0.552} \\ &= 0.7429 = 0.7 \text{ sec} \end{aligned}$$

3. Calculating the relative standard deviation (RSD):

$$RSD = \frac{s}{\bar{x}} \times 100 \quad \text{Equation 3}$$

Where:

s = standard deviation

\bar{x} = the mean of the set of measurements

If $RSD < 5\%$, you were precise in your measurements.

Example 3: Calculate the relative standard deviation for the data set used in example 2:

$$RSD = \frac{0.7429 \text{ sec}}{22.32 \text{ sec}} \times 100 = \underline{3.328} = \mathbf{3\%}$$

4. Calculating the relative percent error:

The relative percent error compares an experimental value to the correct or expected value, expressing the answer as the absolute value of a percentage. A relative percent error of 0% means that the experimental value was the same as the expected value.

$$\text{Relative \% error} = \left| \frac{\text{true value} - \text{experimental value}}{\text{true value}} \right| \times 100 \quad \text{Equation 4}$$

Where: *true value* is the expected or theoretical value

experimental value is the calculated value based on data

Example 4: The expected or true value for the reaction studied in Example 1 is 21.95 sec. Calculate the relative percent error.

$$\text{Relative \% error} = \left| \frac{21.95 \text{ sec} - 22.32 \text{ sec}}{21.95 \text{ sec}} \right| \times 100$$

$$= \left| \frac{-0.37 \text{ sec}}{21.95 \text{ sec}} \right| \times 100$$

$$= |-0.016856| \times 100$$

$$= \underline{1.6586\%} = \mathbf{2\%}$$