E12b: Determining Resistance & Resistivity with a Wheatstone Bridge

Introduction:

The Wheatstone Bridge is a circuit that is designed to make very precise measurements of the resistance of different materials. The Wheatstone Bridge takes its name from the man who first demonstrated its multiple applications: Charles Wheatstone, a 19th century English scientist and inventor who also contributed to the development of the telegraph. The basic design of the bridge circuit is so effective that it has been included in numerous types of precision measurement components, such as transducers and strain gauges; precision instruments used in such modern day technologies as radios, televisions, and computers.

This experiment makes use of a Slide-Wire Wheatstone Bridge, one of several different variations to the Wheatstone Bridge Circuit. Utilizing this bridge it becomes possible to explore the relationships between the resistance of a conductor and the physical properties of the conductor, such as its length and resistivity. With a Slide-Wire Wheatstone Bridge, if several resistors are placed in a circuit, the relationships between the physical properties of the resistors (such as wire length and cross-sectional area) and the resistance they produce can be utilized to find unknown physical quantities, such as the resistivity of the material. Determining the resistivity of several unknown wires represents the focus of this experiment.

Apparatus:

- Slide-Wire Bridge Component
- Sensitive Current Meter
- Power Supply
- Known Variable Decade Resistor component
- Unknown Resistance Coils
- Unknown Length of Wire
- Hook-up wires
- Wire Cutter
- Digital Micrometer

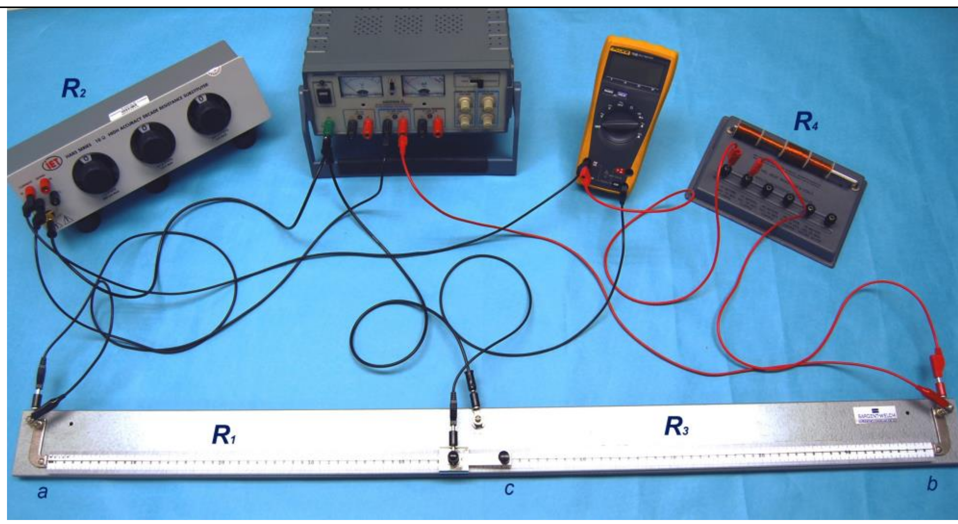

Figure 1

Discussion:

A bridge circuit is any circuit in which the current being sent through a conductor is split into two different parallel paths, and then recombined later in the circuit into a single conductor, thus creating an enclosed loop. There are multiple different designs for bridge circuits that can be constructed with a variety of circuit components, depending upon the intended function of the circuit. The Wheatstone bridge represents the most well known type of bridge circuit.

For this experiment, the specific bridge circuit will be composed exclusively of resistors, and will allow for the measurement of very small resistances. The basic design for a Slide-Wire Wheatstone Bridge circuit of resistors is demonstrated below in **Figure 2**:

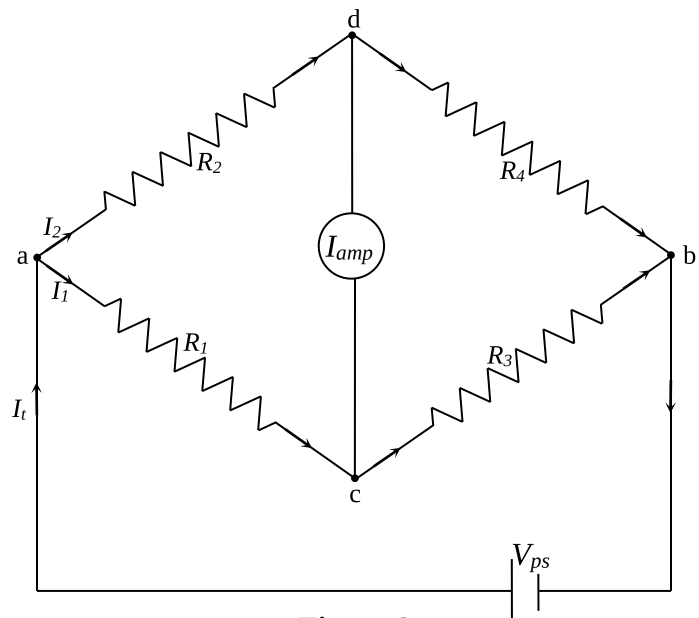

Figure 2

In order for the Slide-Wire Wheatstone Bridge to function correctly, it is necessary to create a balanced bridge. A balanced condition of the bridge exists when no current flows through the meter measuring current (I_{amp}). The balanced condition of the bridge is actually a necessary physical condition for the Wheatstone Bridge to be effective in making precise measurements of resistance. Having a balanced bridge, with no current flowing thru the current meter, allows the derivation of the equations and analysis that enable the Wheatstone Bridge to work successfully.

The first step to deriving the necessary equations and conditions for the Slide-Wire Wheatstone Bridge is the application of Kirchoff's current junction rule and Kirchoff's voltage loop rule:

> ***Kirchoff's current junction rule*** states that: at any point in an electrical circuit where charge density is not changing in time, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.

When the slide-wire bridge is balanced, the electric potentials at point *c* and *d* in **Figure 2** are equal, which signifies that no current is flowing between points *c* and *d*, as indicated by the zero reading obtained via the current meter. Since there is no current moving between these two points, the current I_2 flowing thru R_2 is the same current flowing thru R_4. If the bridge were not balanced, though, it would result in current flowing between point *c* and *d*, which would complicate analysis, resulting in much more complex equations resulting from the application of Kirchoff's voltage loop rule.

> ***Kirchoff's voltage loop rule*** states that: the directed sum of the electrical potential differences around any closed circuit must be zero. This statement is analogous to stating that the algebraic sum of various potential drops across an electrical circuit is equal to the electromotive force acting on the circuit.

Because Kirchoff's voltage loop rule implies that the sum of the voltages around a loop must equal zero, it becomes possible to derive the following equations:

$$I_1R_1 - I_2R_2 = 0$$
$$I_1R_3 - I_2R_4 = 0$$

Solving the two voltage loop rule equations for a ratio between the different currents and then combining the two separate equations provides an expression relating the resistors.

$$\frac{R_2}{R_1} = \frac{R_4}{R_3}$$

This experiment will use the bridge design to measure the resistance of several small unknown resistors, which can then be used to determine the resistivity of those wires. The unknown resistors will be inserted into the resistor R_4 position. Solving the above equation for R_4 gives the following equation:

$$R_4 = R_2 \frac{R_3}{R_1}$$

In the slide-wire form of the Wheatstone Bridge resistors R_1 and R_3 are replaced by a length of uniform wire stretched between two points (*a* and *b*). A sliding contact at point *c* (please consult **Figure 1**) provides a variable resistance ratio between R_1 and R_3. As the sliding metal contact point is moved along the length of the wire, the resistances of R_1 and R_3 can be changed.

The resistance of a uniform wire conductor depends on the material, its length and its cross sectional area. This relationship is expressed in the following equation:

$$R = \rho \frac{length}{area}$$

In this equation *area* represents the cross sectional area of the wire, *length* is the length of the wire, and ρ is the resistivity of the wire material. This allows one to replace the ratio of resistances in the equation for R_4 with the equivalent ratio of lengths, resistivities, and areas.

$$R_4 = R_2 \left(\frac{\rho l_{cb} / area}{\rho l_{ac} / area} \right)$$

This equation can be further simplified by eliminating the resistivity, ρ, and the *area*. Since the apparatus uses a single length of wire to create R_1 and R_3, that wire has a uniform cross-sectional area and a resistivity that remains constant for the length of the wire.

$$R_4 = R_2 \frac{l_{cb}}{l_{ac}}$$

This equation will be used to experimentally determine the resistance of each of the unknown resistors.

Once the experimental resistance of the different unknown resistors has been determined, it then becomes necessary to calculate the experimental resistivity of those unknown resistors. This can be done by making use of the equation expressing the relationship between resistance, resistivity, length, and cross-sectional area given earlier. Solving that equation for resistivity instead of the resistance gives the following formula:

$$\rho_{\exp} = R_{\exp} \frac{area}{length}$$

Table I provides the specific physical properties for the coils used in the experiment. The information for the length of wire that will be used in **Part II** of the experiment has not been provided, and instead all relevant information will need to be determined experimentally.

Table I

Material:	Copper		
Resistivity:	1.678×10^{-8} *ohm m* @ 20^{o} C (*CRC Handbook of Chemistry and Physics*)		
Coil #	**Gauge**	**Diameter *(m)*** (±3%)	**Length *(m)*** (± 5%)
1	22	6.44×10^{-4}	10.00
2	28	3.21×10^{-4}	10.00
3	22	6.44×10^{-4}	20.00
4	28	3.21×10^{-4}	20.00

With the decade resistor component, R_2 in this experiment, it is possible to maintain an extreme degree of accuracy in varying the known resistance R_2 when finding the unknown resistance R_4. Because of this accuracy in controlling the known resistance, which can be set to the hundredth of an ohm (Ω), it becomes possible to center the search for the lengths *ac* and *cb* around the center of the uniform wire that creates R_1 and R_3 in **Figure 1**. The reason for remaining near the center (*50.0 cm*) position of the uniform length of wire is to increase the accuracy of ratio $\left(\frac{R_3}{R_1}\right)$. If the ratio between resistances or lengths is extreme (75/25 or 90/10 for example), then positioning or measurement errors can drastically decrease the precision of the experiment.

Please center the search ±*10.0 cm* for the lengths *ac* and *cb* around the *50.0 cm* position, varying the known resistor to find the balance point, rather than moving the contact point along the entire length of the wire.

Procedures:

This lab has been divided into two different sections, with the goal of each section being to determine the resistivity of several wires of varying lengths and different materials. **Part I** focuses on several coils of wire with known lengths and diameters and also with a theoretical resistivity available for comparison. In **Part II,** the length, area, and resistivity of the wire will be unknown, making it necessary to measure the wire's length and diameter before finding the resistance and ultimately calculating the wire's resistivity. **Part II** will also require multiple trials be taken at each length, as the wire is cut down according to the specifications in this **Procedures** section.

Part I

1. Confirm the apparatus is setup as shown in **Figure 1**.
2. Connect the first unknown wire coil to the slide-wire bridge in the R_4 position. This should mirror the setup in **Figure 1** completely. For all subsequent trials, please be aware that the R_4 unknown resistance coils are set up in series, such that the second unknown resistance coil would be connected by shifting the wires one position to the right.
3. Next, turn on the power supply. **DO NOT ADJUST THE POWER SUPPLY IN ANY WAY**. Increasing the voltage could potentially damage the known decade resistor component. **The current must remain below 200 milliamps**.
4. Select an appropriate value for the known resistance R_2 on the decade resistor. In general, it is recommended that the decade resistor should be set to 1.00 Ω resistance as the starting position for each search, and then varied slightly up or down in resistance.
5. Move the contact point (c) along the slide-wire, making intermittent contact with the wire until the position is found where the current meter indicates zero when point (*c*) is in contact with the slide-wire. This indicates that the bridge is balanced. If the bridge cannot be balanced between the *40.0 cm* and *60.0 cm* positions at 1.00 Ω, try increasing and decreasing the known decade resistance by different increments until a balanced bridge (where the current meter indicates 0) becomes possible.
6. Measure l_{ac} & l_{cb} and calculate the experimental unknown resistance.
7. Calculate the resistivity of wire material for the coil.
8. Compare the theoretical and experimental resistivity values. If they are not similar check the bridge connections. Also examine if a different known resistor permits a better experimental determination. It is possible to find numerous different lengths at different resistances that balance the bridge.
9. Repeat the process using the other unknown wire coil resistors.

Part II

1. Obtain one piece of the unknown wire from the lab instructor, along with suggestions for how to properly connect this unknown length of wire to the bridge circuit. Throughout the experiment it will be necessary to conduct three different trials for five different lengths of wire. After determining the lengths and resistances for three different trials at one specified length, the wire will need to be cut to a shorter length before proceeding to the next set of measurements.
2. Precisely measure the diameter of the wire using the digital micrometer and calculate its cross sectional area. This calculation may be assumed to hold for the entire length of this unknown wire.

Part II (cont.)

3. Attach the full length of wire to the apparatus as suggested by the lab instructor. In general, it is suggested that the wire length be inserted into the two final terminals of the unknown resistors connected in series. The wire should be inserted so that it is touching the bottom of each terminal, and the hook-up wires should be inserted as well, creating a connection between the two wires and the slide-wire bridge apparatus. Please reference **Figure 3** for an example of how this connection should look.
4. Find an appropriate known resistor that permits a balanced bridge position as close to *50.0 cm* as possible. Whereas in **Part I** it was acceptable to find the contact point (*c*) within a range of ±*10.0 cm* of the *50.0 cm* point, for this first trial it will be necessary to find a contact point (c) that is ±*0.5 cm* of the *50.0 cm* mark. Balance the bridge by repeated experimental testing and adjustment of the known decade resistor R_2 and the contact point.
5. Record the value of the known resistor R_2 and the lengths l_{ac} & l_{cb}. Calculate the value of the unknown resistance R_4.
6. Repeat this process (steps **4.** and **5.**) to find a balanced position between *42.0cm* and *48.0 cm*.
7. Again repeat this process (steps **4.** and **5.**) to find a balance position between *53.0 cm* and *57.0cm*. This will complete the three measurements for this length of wire.
8. Mark the contact position on each end of the wire. To do so, please use the tip of the wire cutters, as shown in **Figure 3**. This will create a clean, easy-to-read crimp, making it possible to more accurately measure the effective length of the unknown wire serving as a resistor.

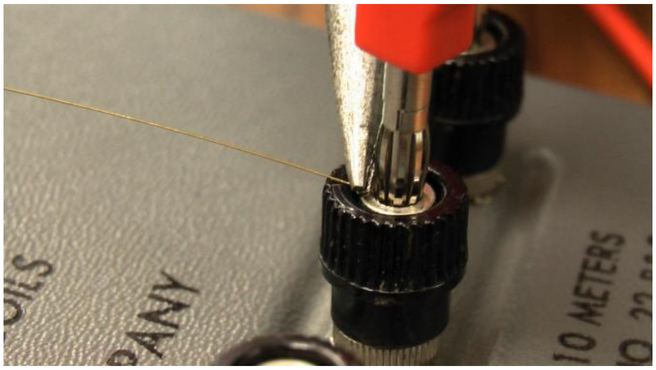

Figure 3

9. Once the wire has been properly crimped to show its effective length, remove it from the connectors. Precisely measure the effective length of the wire. It is important to be as accurate as possible, measuring only the length between the two bends in the wire created by the wire cutters in step **8.**
10. Now please cut *6.0 cm* from each end of the wire, shortening it to a new length.
11. Repeat steps **5.** thru **10.** for this new wire length. This process will need to be repeated for a total of five wire lengths.
12. Construct a linear graph using all 15 data points. It is recommended that the graph be constructed in MS Excel, to allow the graph be printed (outside of this physics lab) for inclusion with the lab report. The wire length should fall on the x-axis of the graph, and the product of the resistance (R_4) and cross sectional area on the y-axis. Determine the slope and the standard deviation of slope.
13. The slope is the resistivity of the wire. Record the value on the **Data Sheet**, making sure to include the appropriate units for resistivity. Compare the resistivity obtained to the known resistivity for this type of metal wire.

Experiment : E12b: Resistance & the Slide-Wire Wheatstone Bridge

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date___________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheet: E12b: Resistance & the Slide-Wire Wheatstone Bridge

NAME: ______________________________ DATE: __________________

Part I

coil #	cross sectional area (m^2)	known resistor R_2 (*ohms*)	length l_{ac} (*m*)	length l_{cb} (*m*)	experimental resistance R_4 (*ohms*)	experimental resistivity ρ (*ohm* m*)
1						
2						
3						
4						

Part II

Wire: diameter *(m)* _____________ **cross sectional area** *(m^2)*________________

Trial	wire length (*m*)	known resistor R_2 (*ohms*)	*length* l_{ac} (*m*)	*length* l_{cb} (*m*)	experimental resistance R_4 (*ohms*)
Length 1					
Length 2					
Length 3					
Length 4					
Length 5					

Resistivity (*Slope*): ______________________ Standard Deviation: ________________