E12c: Kirchhoff's Rules and Bridge Circuits

Introduction:

In this experiment, Kirchhoff's junction and loop rules will be used to analyze a multi-loop circuit. In part I, the goal is to observe the validity of Kirchhoff's rules by measuring voltages across and currents through each element of a multi-loop circuit. In part II, the goal is to learn how to determine the resistance and resistivity of a wire with high precision using a Wheatstone bridge circuit. For this purpose, the bridge circuit will be reconfigured with a precision decade resistor and an ammeter to determine the unknown resistances of three wires.

Apparatus:

- Circuit Breadboard
- Sensitive Multimeter
- Power Supply
- Variable Decade Resistor
- 5 Resistors, 10 to 2000 Ohm
- Hook-Up Wires & Connectors
- *Three segments of wires*
- Digital Micrometer
- Meter stick

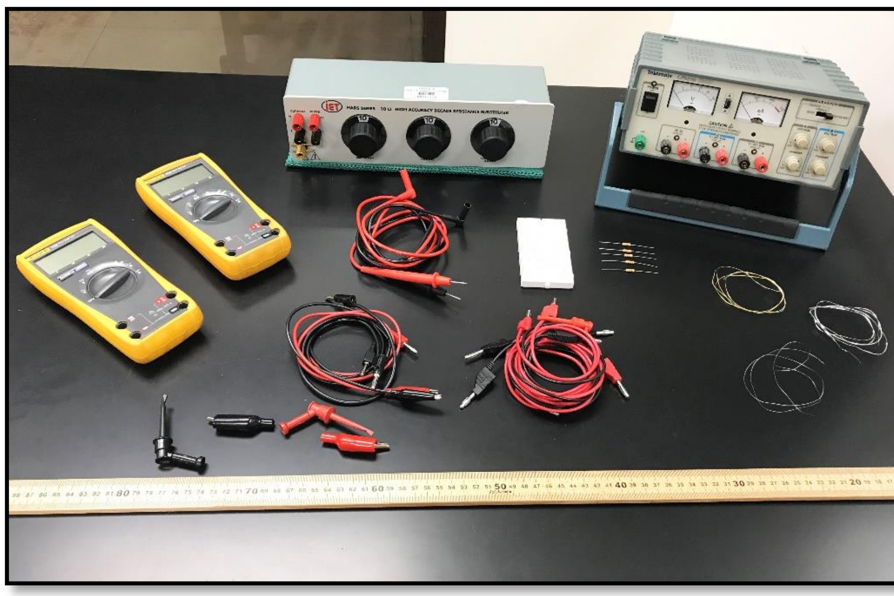

Figure 1. Experimental apparatus.

Theoretical Background:

I. Kirchhoff's Rules

Kirchhoff's rules follow from the principles of conservation of charge and conservation of energy and govern the behavior of all electric circuits.

A simple DC circuit, consisting only of series and parallel combinations of ohmic resistors, can be analyzed by finding the equivalent resistance and applying Ohm's law. However, in general, circuits not always can be split on series and parallel combinations. Circuits can contain non-ohmic elements, like diodes and thermistors; they often include capacitors and inductors and are driven by an alternating (AC) power supplies. Kirchhoff's rules give us a way to find the overall behavior of any electrical circuit.

Kirchhoff's Junction (Node) Rule reflects conservation of charge and states that *the sum of all currents entering a junction (node) equals the sum of all currents leaving this junction.* The simple bridge circuit you will explore in this lab has four junctions (nodes), labeled *a–d* in Figure 2.

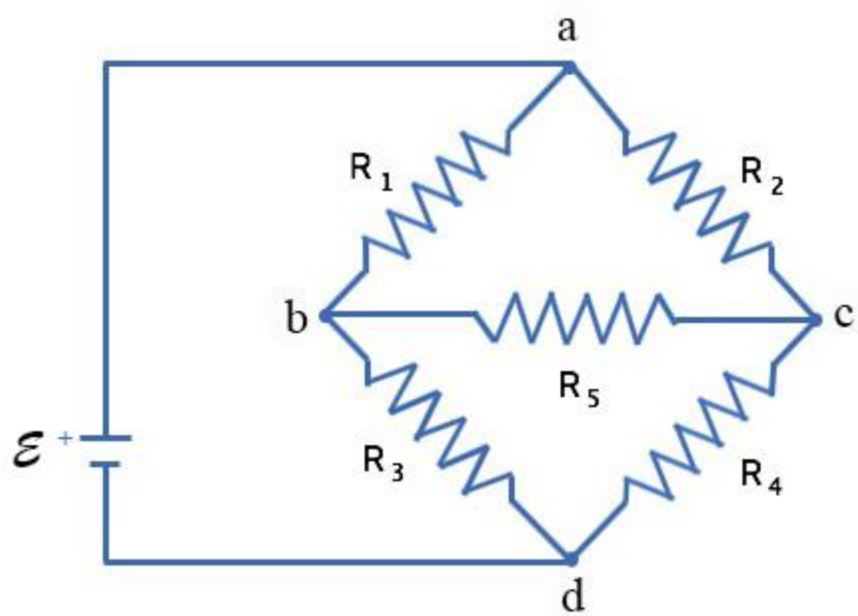

Figure 2. Simple bridge circuit with current junctions (nodes) *a*, *b*, *c*, and *d*. Since the internal resistance of the power supply is negligible, the potential difference or voltage across the power supply, $V_B = \varepsilon$.

To observe the validity of the *junction (node) law* in the lab, you will use an ammeter to measure the magnitude and direction of the current through each resistor. Instructions are provided in the procedures.

Kirchhoff's Loop Rule reflects the principle of conservation of energy and states *that sum of all potential differences (voltages) across any closed loop in a circuit must equal zero*. The circuit shown in Figure 2 has seven different loops.

The potential difference across the terminals of a battery (or a DC power supply) is $V_B = \varepsilon - Ir$, where ε is electromotive force, I is current through the battery and r is internal resistance of the battery. If r is negligible, then $V_B = \varepsilon$.

The potential difference across a resistor, V_R, equals the current through the resistor, I_R, multiplied by the resistance, R, of the resistor: $V_R = I_R R$.

One can add voltages around any chosen loop and either direction: clockwise or counterclockwise. However, consistency with the sign (decrease or increase in electrostatic potential) is important. The sign conventions for potential differences when Kirchhoff's loop law are applied to a circuit loop can be stated as follows:

1. Current always flows in the direction of a decreasing potential energy. Therefore, if voltage across a resistor is counted in the direction of current, the potential difference must be negative ($-I_R R$). If voltage is counted in the opposite direction of the current, the potential difference must be positive ($+I_R R$).

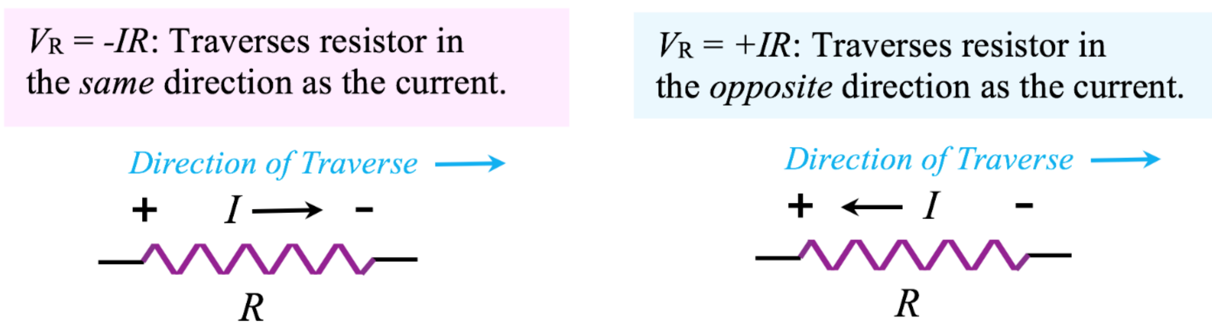

Figure 3. The direction over which the resistor is traversed affects the sign of the potential difference.

2. The positive terminal of a battery (or DC power supply) is always at a higher potential than its negative terminal. Therefore, if voltage is counted from positive to negative terminal, it must be taken as negative ($-\varepsilon$). If voltage is counted from negative to positive terminal, it must be taken as positive ($+\varepsilon$).

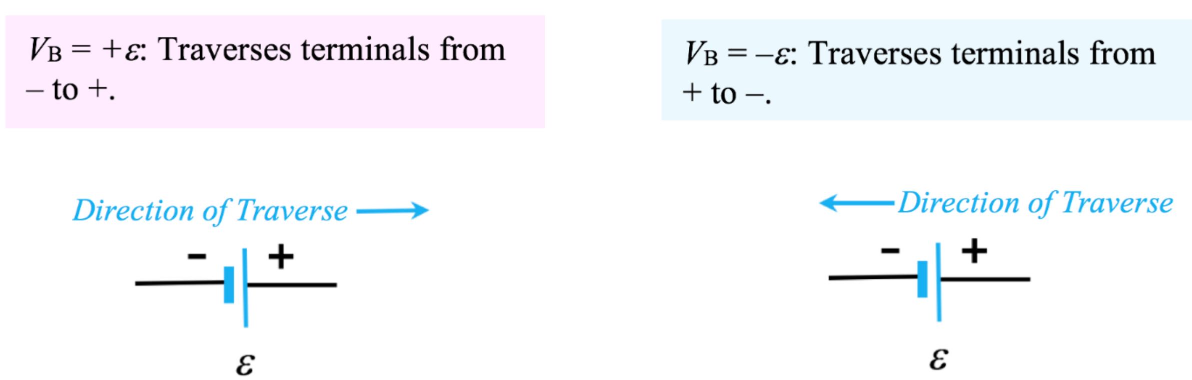

Figure 4. The direction over which the battery or power supply is traversed affects the sign of the potential difference.

To observe validity of the loop rule in the lab, you will need to know the direction of the current in each resistor and the polarity of the power supply.

II. The Wheatstone Bridge

A *bridge circuit* is a circuit in which the current splits into two parallel paths only to recombine into a single current elsewhere in the circuit, thus creating a separate loop. There are many types of bridge circuits that can be constructed with a variety of components, depending upon its intended function.

The Wheatstone bridge is a well-known type of bridge circuit. It is designed to precisely measure small values of resistance. The Wheatstone Bridge takes its name from Charles Wheatstone, a 19th century English physicist and inventor who also contributed to the development of the telegraph. The basic design of the bridge circuit is so effective that it has been included in numerous types of precision measurement components, such as transducers and strain gauges; precision instruments used in such modern day technologies as radios, televisions, and computers.

For this experiment, the bridge circuit will be composed exclusively of resistors and will allow for precision measurements of the resistance of several wires. The basic design for a Wheatstone bridge circuit is demonstrated below in Figure 5:

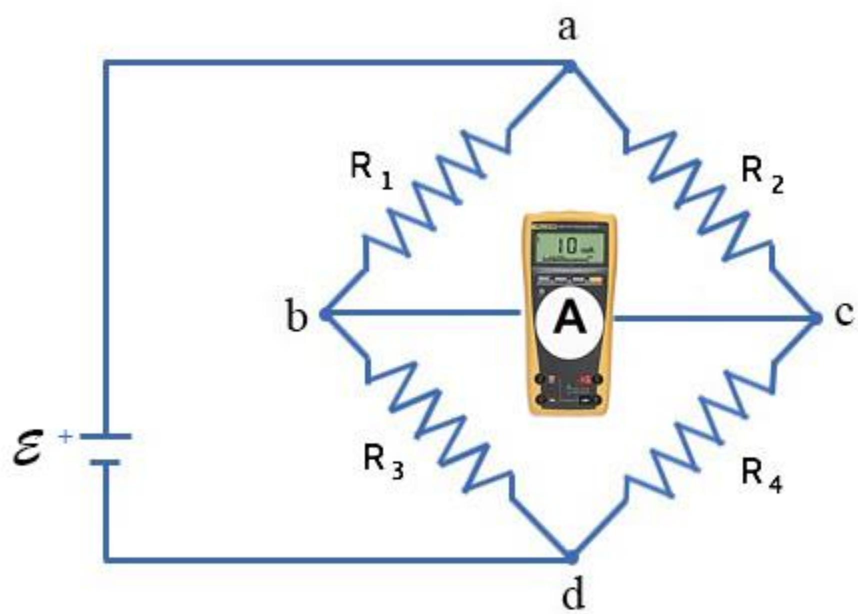

Figure 5. The Wheatstone Bridge.

In order to determine an unknown resistance, it is necessary to create a *balanced* bridge. A bridge circuit is *balanced* when no current flows through the ammeter (**A**).

We begin by applying Kirchhoff's junction (node) rule and Kirchhoff's loop rule:

When the bridge is balanced, the electric potentials at points *b* and *c* in Figure 5 are equal since no current flows between them, as indicated by the zero reading obtained on the ammeter. Since there is no current between *b* and *c*, the current I_2 (through R_2) must be the same current as I_4 (through R_4) and the current I_1 (through R_1) must be the same current as I_3 (through R_3). If the bridge were *unbalanced*, meaning current flows between points *b* and *c*, the precise measurement of the unknown resistor would be quite difficult.

Applying Kirchhoff's loop law to loop *abc* and to loop *bcd* (Figure 5) with the bridge *balanced* gives the following equations:

$$I_1R_1 - I_2R_2 = 0 \qquad (1)$$

$$I_1R_3 - I_2R_4 = 0 \qquad (2)$$

Eliminating I_1 and I_2 from (1) and (2) gives:

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} \qquad (3)$$

You will use this bridge design to measure the precise resistance of several small wires, which can then be used together with accurate wire length and wire diameter measurements to determine their resistivities.

Resistivity, ρ, is an intrinsic property of a material, which is defined as a ratio of the electric field in the material, *E*, to the current density, *J*, it generates in this material:

$$\rho = \frac{\vec{E}}{\vec{J}} \qquad (4)$$

The resistance of a circuit element with constant cross sectional area, *A*, and length, *L*, is then given by:

$$R = \rho\frac{L}{A} \qquad (5)$$

In the experiment, wires of unknown resistance will be inserted into the position of R_2. The bridge will be balanced by changing the value of decade resistor, which is placed in the position of R_4. Then, the unknown resistance R_2 can be calculated using equation (3).

Once the resistance of a wire is calculated, its resistivity can be determined by measuring its length and its diameter and then by using equation (5).

For additional information on these concepts please read/review the following sections in your textbook.

Kirchhoff's Rules:

	Young & Freedman.	**Sears & Zemansky's University Physics,** *Chapter* **26** *section* **2**
	Walker.	**Halliday & Resnick Fundamentals of Physics,** *Chapter* **27** *section* **7**

Resistance & Resistivity:

	Young & Freedman.	**Sears & Zemansky's University Physics,** *Chapter* **25** *sections* **2** & **3**
	Walker.	**Halliday & Resnick Fundamentals of Physics,** *Chapter* **26** *section 4*

Procedures:

Part I Wheatstone Bridge and Kirchhoff's Rules

1. Determine the color coded values of the five resistors provided and record them in Table 1.
2. Measure the actual resistance of each resistor with a multimeter and record it in Table 1.
3. Assemble the five resistors in a bridge combination on the breadboard as is shown in Figure 2. Label the resistors on **Diagram 1** of your data sheet section. Connect the bridge to a power supply as shown in Figure 2. Ask your lab instructor to check your circuit.
4. Turn on the power supply and set its voltage to approximately **5.00 V**. Measure and record the set voltage in Table 1. Do not change the setting during the experiment.
5. Using a multimeter in voltmeter mode, measure the potential differences across each resistor. Record the measured values in Table 1.
6. Using a multimeter in **voltmeter** mode, determine the direction of current in each resistor. *Hint: potential difference is decreasing in the direction of current; therefore, if your voltmeter is set correctly, and it gives positive values for voltage across a resistor, the current in this resistor is flowing from the red probe to the black probe of the voltmeter. If your values for voltage are negative – current flows from the black probe to the red probe*. Label the direction of the currents on **Diagram 1**. Ask the instructor to check your labeled directions of currents.
7. Using a multimeter in ammeter mode, measure currents through each resistor and the total current in the circuit. Record the measured values in Table 1.
8. Identify seven loops in the circuit. Highlight the loops in the **Diagrams 2-8** (one loop per a diagram) and indicate the chosen direction of the loop. **Use your measured voltages** to confirm that the sum of potential differences across each loop is zero (Kirchhoff's loop rule). Show your calculations in the space to the right of each appropriate diagram.
9. Identify four junctions of the circuit. Mark these junctions on diagrams 9-12. Use your measured currents to confirm that for each junction the sum of currents entering the junction equals the sum of currents leaving the junction (Kirchhoff's junction rule). Show your calculations in the space to the right of each appropriate diagram.
10. Optional: Using the set value of voltage of the power supply and measured values of the resistors, calculate the theoretical values of currents in each resistor. Compare them with your measured values.

Part II Resistance and Resistivity of Unknown Wires

1. For each wire, use the meter stick to measure the length and the digital micrometer to measure the diameter. Record the measured values in Table 2.
2. Make sure the power supply is off and then, in your assembled bridge, substitute into the middle resistor, position R_5, an ammeter as in Figure 5. Place the 100 Ω resistor into the position R_1. Place the 240 Ω resistor into the position R_3. Place the variable decade resistor

in the position R_4. Place one of your unknown wires in the position for R_2 as in Figure 5. Ask the instructor to check your circuit.

3. Turn on the power supply and set its maximum current to 200 mA. Be careful as to not exceed 200 mA and overload the circuit. Adjust the current throughout the experiment as needed.
4. Adjust the resistance of the decade resistor until there will be no current detected through the ammeter. Record the decade resistor value you found in Table 2.
5. Turn off the power supply and replace your first unknown wire with the second unknown wire. Repeat steps 3 and 4.
6. Turn off the power supply and replace your second unknown wire with the third unknown wire. Repeat steps 3 and 4.
7. Turn off the power supply. Use Kirchhoff's loop rule to establish a relationship between the unknown resistance of the wire (R_2), and resistors R_1, R_3, and R_4 (decade resistor). Calculate the resistance of each wire.
8. Calculate the resistivity of each wire.
9. Check your resistivity values with the lab instructors to verify they are within normal parameters.

Experiment E12c: Kirchhoff's Rules and Bridge Circuits

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheet: E12c: Kirchhoff's Rules and Bridge Circuits

NAME: ____________________________ DATE: _________________

Part I

Voltage of power supply V_B_______________________

Total current I_t_________________________________

Table 1

Resistor #	Color Coded Resistance (Ω)	Measured Resistance (Ω)	Potential Difference (V)	Current (mA)

Part II

R_1 = _____________

R_3 =______________

Table 2

Wire #	Decade Box Resistance (Ω)	Resistance of the wire (Ω)	Length of the wire (m)	Diameter of the wire (m)	Cross Sectional Area (m^2)	Resistivity of the wire (Ω m)

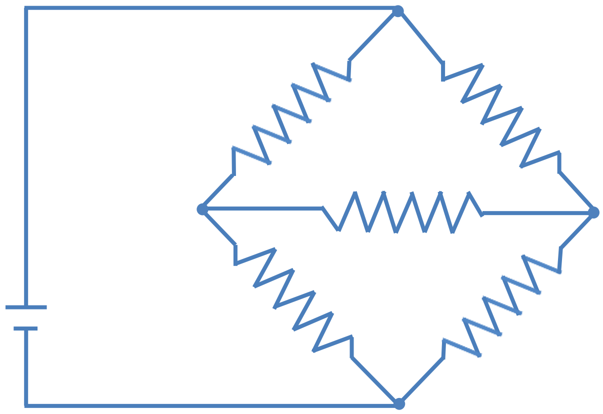

Diagram 1

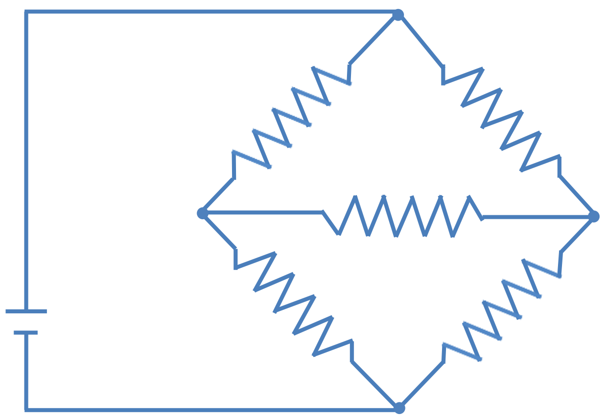

Diagram 2

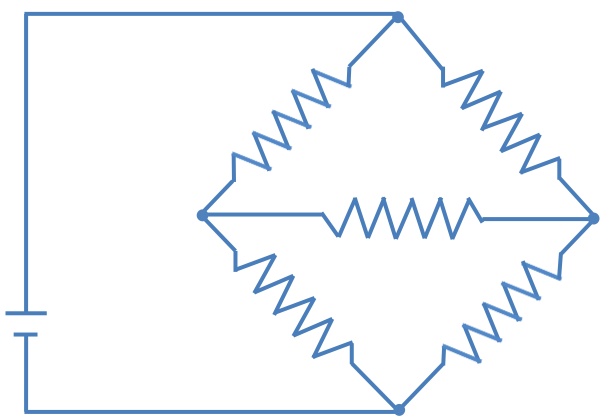

Diagram 3

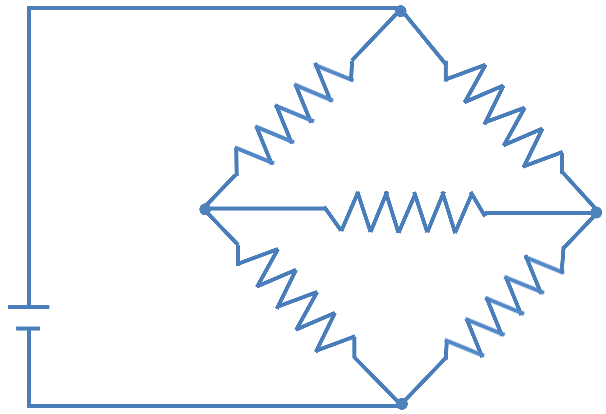

Diagram 4

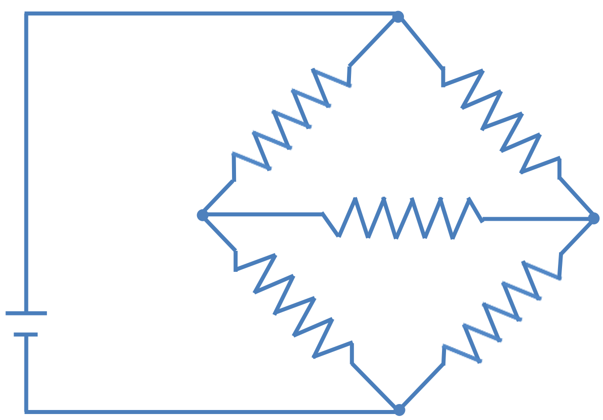

Diagram 5

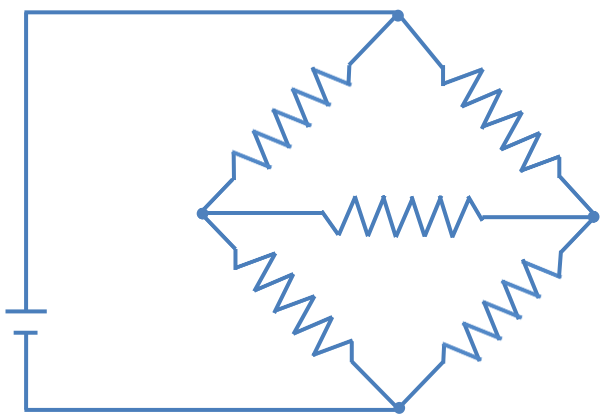

Diagram 6

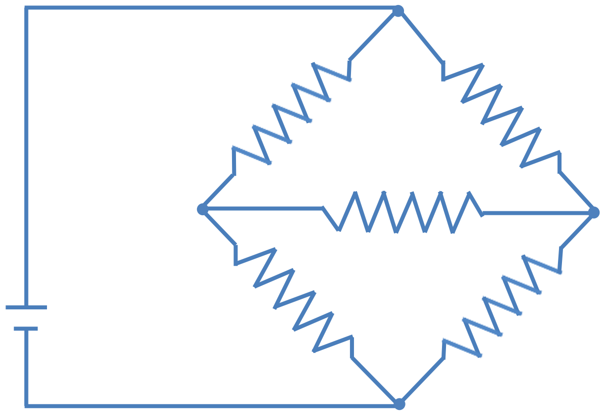

Diagram 7

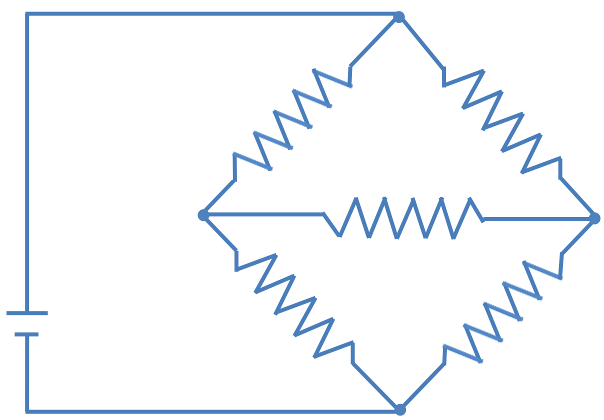

Diagram 8

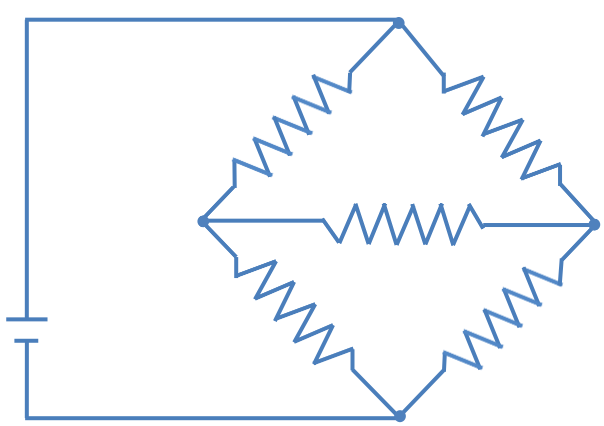

Diagram 9

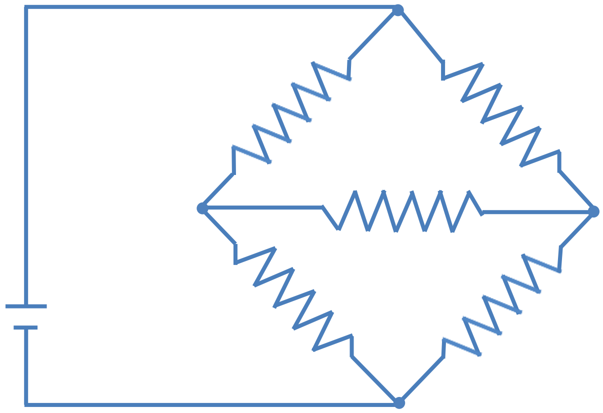

Diagram 10

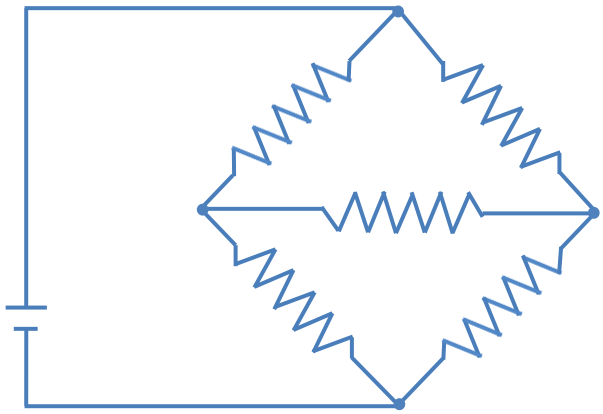

Diagram 11

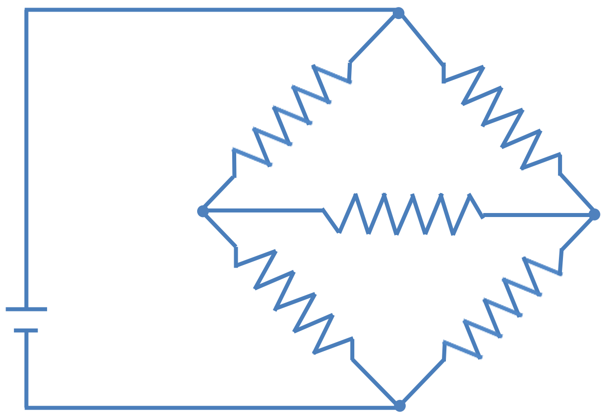

Diagram 12

Questions:

1. Does the resistivity of a material depend on its length or cross-sectional area? Does resistance depend on the length or cross-sectional area? Explain?

2. Explain why the sum of the changes in potential around a closed loop is zero.

3. Explain why the sum of the currents into and out of a node is zero.

4. Is it possible to find the value of an unknown resistor in a bridge circuit without using Kirchhoff's Rules? (For example, could you use parallel and series combinations of resistors to determine the total resistance and compare that to V/I?) Explain your answer.

5. Suppose one of the resistors in the bridge circuit was heated by baking it in an oven. How would this effect the current flowing through it after placing it back into the circuit?