M23b: Rotational Dynamics & Determining the Moment of Inertia

Introduction:

This experiment examines rotational dynamics and the properties of moment of inertia. In its linear form Newton's Second Law establishes the relationship between mass, net force and the resulting acceleration. For rotation Newton's Second Law in its rotational form establishes the same kind of relationship between the moment of inertia, net torque and angular acceleration. The moment of inertia expresses not only how much mass an object has but also how this mass is distributed with respect to the axis of rotation.

The main purpose of this experiment is to determine the moment of inertia of a hoop both theoretically and experimentally. The hoop will be resting on a platform composed of a large irregular disk, sitting on top of a small step pulley which is attached to a rotating shaft. The shaft is mounted to a heavy solid base using bearings. Before determining the moment of inertia for the hoop it is necessary to determine the moment of inertia for the platform. But given the irregularities in the disk's surface and how it's mounted on a rotating shaft with all of the other components this can be calculated only approximately with a formula. Therefore the moment of inertia for the platform will be obtained experimentally. Once the experimental moment of inertia of the platform is known, the experimental moment of inertia of the hoop can be obtained. This value will be compared against the theoretical moment of inertia for the hoop, which can easily be calculated with a formula. Additionally as an approximation, the theoretical moment of inertia for an ideal, uniform disk having the same general dimensions and mass like the one that is part of the platform will be calculated and compared to the experimental value obtained for the platform.

Apparatus:

- Height-adjustable stand with a bearing mounted rotating shaft and attached step pulley
- Large Disk and Hoop
- Rod mounted Pulley
- Mass Hanger, Masses and String
- Mass Scale
- Digital Caliper
- Ruler
- Photogate with Computer timing system

Figure 1

Discussion:

The dynamical behavior of rotation has analogous components to the behavior of translational motion. Torque replaces force, angular acceleration replaces linear acceleration and moment of inertia replaces mass. The moment of inertia of a body describes not only how much matter a body is composed of, but also how that matter is distributed about the axis of rotation. This experiment examines some of the properties associated with these components of rotation. When a rigid body is acted upon by a system of torques, where the sum of these torques is zero, the rigid body is in equilibrium with respect to rotation. This means that the body can only have two rotational motion states: to be at rest or to rotate uniformly about a fixed axis. If the sum of the system of torques is not equal to zero but is equal to some net torque, then the rigid body will experience an angular acceleration in the direction of the net torque. This behavior is described in Newton's second law for rotation:

$$\tau_{net} = I\,\alpha$$

Here τ is the torque, $\boldsymbol{\alpha}$ is the angular acceleration and $\boldsymbol{I}$ is the moment of inertia. Again, when the net torque is not equal to zero, the rigid body experiences an angular acceleration in the direction of the net torque.

The mass, usually used in the linear expressions, doesn't have enough information for rotation. It must be replaced by the moment of inertia of the body. This quantity describes where each particle of mass is located relative to the axis of rotation. It is a geometric characteristic of the object, as it depends only on its shape, its mass distribution throughout the shape and the position of the rotation axis. In its discrete form it is defined by:

$$I = \sum_{i=1}^{N} m_i r_i^2$$

This expression means take each particle of mass multiplied by its radius squared (where the radius is the distance from the center of the mass to the center of the axis of rotation) and add them all up to get the total.

$$\boldsymbol{I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 \dots m_N r_N^2}$$

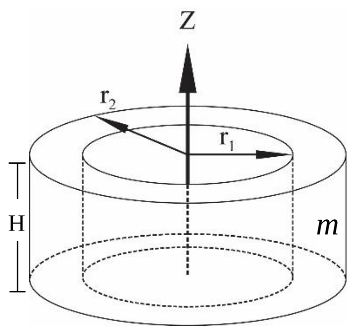
For a solid body this discrete form calculation becomes too complicated and is transformed into the continuous form:

$$\boldsymbol{I = \int r^2\, dm}$$

This is also frequently represented in a three dimensional form using ρ as the density and dV as the volume element:

$$\boldsymbol{I = \iiint r^2 \rho\, dV}$$

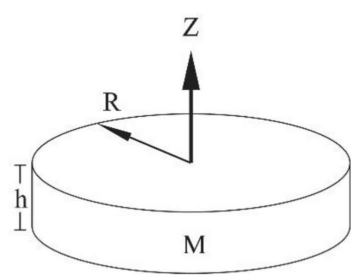
For a thick-walled cylindrical tube with open ends, inner radius ($\boldsymbol{r_1}$), outer radius ($\boldsymbol{r_2}$) and mass (m) rotating about (z) axis, the moment of inertia can be calculated by:



$$I_z = \frac{1}{2} m(r_1^2 + r_2^2)$$

Figure 2

For a disk with radius (R) and mass (M) rotating about (z) axis, the moment of inertia can be calculated by:

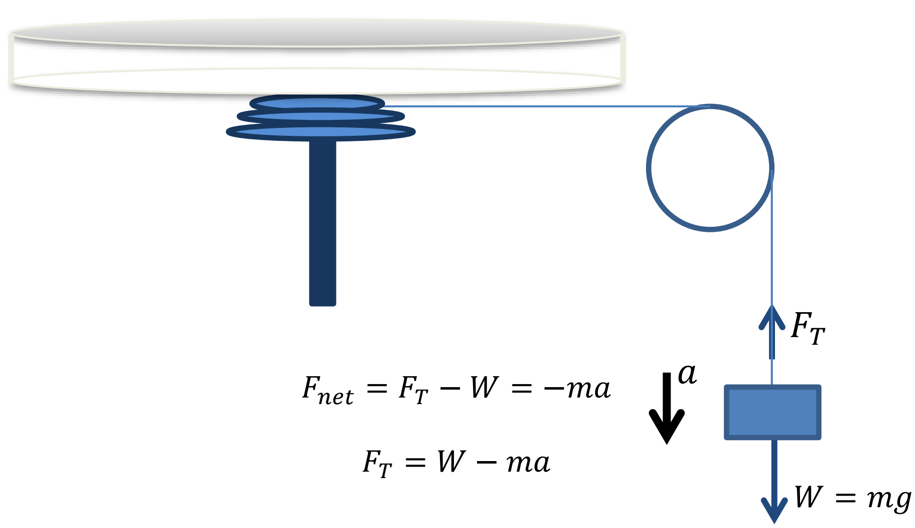


$$I_z = \frac{1}{2} MR^2$$

Figure 3

The rotation in this experiment is produced by an external applied torque. The force for this torque is from the tension in an attached string ($\boldsymbol{F_T}$) due to a mass hanging on the other end. The weight of the mass is greater than the tension in the string and results in the mass moving with acceleration (a) downward. The magnitude of the tension can be calculated by the following:

$$F_T = m(g - a)$$



$$F_{net} = F_T - W = -ma$$

$$F_T = W - ma$$

Figure 4

Procedures:

This experiment consists of several sets of trials. The first set focuses on varying the radius where the force is applied and thereby varying the applied torque. The second set of trials focuses on varying the applied force in order to again vary the applied torque. In all cases the torque is being applied to either the platform alone or the platform with hoop added, via the step pulley mounted underneath on the rotational shaft.

1. The step pulley *under* the disk has three different radii. Measure the diameter of each step where the string will be wound and then calculate each radius.
2. For the large Disk measure its mass **using a special balance for heavy masses** (ask a lab instructor to identify the location of this balance) and diameter using a ruler.
3. For the thick Hoop measure the inside and outside diameters using a caliper as well as its mass **using a special balance for heavy masses** (ask a lab instructor to identify the location of this balance).

Varying the Torque via Changing the Radius

4. Add 50-grams to the mass hanger and measure the total mass. This mass will remain constant during this part.
5. Wind the string on the largest radius until the mass hanger is suspended close to the outside pulley.
6. Adjust the external pulley so that the string is aligned with the pulley rod.
7. Follow the provided computer instructions to begin the computer data collection. The computer will graph angular velocity over time. Obtain the *linear regression line* to find the *angular acceleration* for each trial. Note: start collecting data first and then release the mass.
8. Obtain the angular acceleration and standard deviation for each trial.
9. Repeat these steps (4-8) but using the other two radii on the step pulley.
10. Next add the hoop on top of the platform, centered, and repeat this sequence of steps (4-9).

Varying the Torque via Changing the Applied Force

11. As applied mass, use 60-grams plus the mass hanger and measure the total mass with the corresponding balance.
12. Wind the string on the medium radius. This radius will remain constant during this part.
13. Conduct the trial as before, obtaining the angular acceleration and its standard deviation.
14. Conduct four additional trials, increasing the mass by 20 grams each time.
15. Next add the hoop on top of the platform, centered, and again repeat the sequence of steps to collect the angular accelerations for each trial.

Analyses:

1. Calculate the linear acceleration, also may be referred to as the tangential acceleration, using the angular accelerations collected.
2. Calculate the tension in the string applied to the step pulley for each trial.
3. Calculate the torque for each trial.
4. Utilize all of the data for the Platform trials and graph the angular acceleration as a function of torque using Excel.
5. Utilize all of the data for the Platform with Hoop trials and graph the angular acceleration as a function of torque using Excel.
6. Obtain the statistical slope and standard deviation for each graph. Determine the experimental moment of inertia for each using the slope from the graphs.
7. Calculate the difference between the moment of inertia for Platform and the Platform with Hoop to determine the experimental moment of inertia for the Hoop alone.
8. Calculate the theoretical moment of inertia for the Hoop.
9. Calculate the percent error between the experimental and theoretical moment of inertia for the Hoop.
10. Calculate the theoretical moment of inertia for the ideal, uniform Disk.
11. Calculate the percent difference between this approximate theoretical value for the Disk and the experimental value you obtained for the Platform.

Experiment M23b: Rotational Dynamics & Determining the Moment of Inertia

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheet 1: M23b: Rotational Dynamics & Moment of Inertia

NAME: ____________________________ DATE: _________________

Platform	**Varying the Torque via Changing the Radius**		
Applied Mass	Radius	Angular Acceleration	Standard Deviation
Platform with Hoop	**Varying the Torque via Changing the Radius**		
Applied Mass	Radius	Angular Acceleration	Standard Deviation
Platform	**Varying the Torque via Changing the Applied Force**		
Applied Mass	Radius	Angular Acceleration	Standard Deviation
Platform with Hoop	**Varying the Torque via Changing the Applied Force**		
Applied Mass	Radius	Angular Acceleration	Standard Deviation

Data Sheet 2: M23b: Rotational Dynamics & Moment of Inertia

NAME: ____________________________ **DATE: _________________**

(Calculations)

Platform	**Varying the Torque via Changing the Radius**	
Linear Acceleration	Tension	Torque

Platform with Hoop	**Varying the Torque via Changing the Radius**	
Linear Acceleration	Tension	Torque

Platform	**Varying the Torque via Changing the Applied Force**	
Linear Acceleration	Tension	Torque

Platform with Hoop	**Varying the Torque via Changing the Applied Force**	
Linear Acceleration	Tension	Torque

Data Sheet 3: M23b: Rotational Dynamics & Moment of Inertia

NAME: ____________________________ DATE: _________________

Experimental Determination Moments of Inertia:

Moment of Inertia for the Platform: __________________

Moment of Inertia for Platform with Hoop: __________________

Moment of Inertia for the Hoop: __________________

Theoretical Determination:

Inside Diameter of the Hoop: __________________

Outside Diameter of the Hoop: __________________

Mass of the Hoop: __________________

Theoretical Moment of Inertia of the Hoop: __________________

Percent Error: __________________%

Diameter of the Disk: __________________

Mass of the Disk: __________________

Theoretical Moment of Inertia of the Disk (Approximate): __________________

Percent Difference: __________________%