M25a: Rotational Equilibrium and Rotational Dynamics

Introduction:

By Newton's First Law of Motion, a body is in **translational equilibrium** when the net force on the object is equal to zero. However, a net force of zero can still allow an object to rotate. The tendency of a force to rotate a body about an axis is called **Torque**. Therefore when examining a body that could rotate and again applying Newton's First Law, the body is in **rotational equilibrium** when the net torque is equal to zero. For translational equilibrium the body will be either not moving or moving in a straight line at constant velocity. Similarly for rotational equilibrium the body is either not rotating or is rotating with constant angular velocity. If the body is completely stationary, meaning both not moving along a line and not rotating, then it is in **static equilibrium**.

Newton's Second Law of Motion explains what happens when the sum of the Forces and/or the sum of Torques are not equal to zero. In these cases the body experiences acceleration as a result of the net force and/or an angular acceleration as a result of the net torque. The Second Law, in rotational form, establishes the relationship between the net torque, the moment of inertia and the angular acceleration. **Moment of Inertia** represents the mass for rotation. It refers not only to how much mass an object has but also to how this mass is distributed with respect to the axis of rotation.

The main purpose of this experiment is to examine the properties of torque, the dynamics of rotation that occur as a result of a net torque and the moment of inertia for a rigid body. First, multiple torques will be applied to a meter stick, putting it in static equilibrium. Then, the individual torques will be determined and used to verify the first law condition for equilibrium. For the dynamic part of the experiment torque will be applied to an apparatus composed primarily of a large horizontal solid disk and a heavy thick hoop that will mount on top of the disk. The disk or disk & hoop combination will accelerate angularly due to the net torque. The data collected will be used to determine the moment of inertia for both bodies. This experimentally determined moment of inertia will be compared to the theoretical value expected.

Apparatus:

Figure 1

Part One:

- Meter stick
- Support stand
- Clamp to hold the meter stick
- Set of masses
- 2 Mass hangers
- Unknown mass

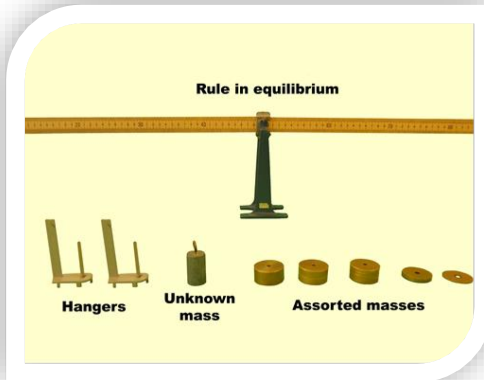


Figure 2

Part Two:

- Height-adjustable stand with center mounted bearing shaft
- 3-step pulley
- Large solid disk
- Heavy thick hoop
- Pulley with mounting rods
- Mass hanger, masses and string
- Caliper and metric ruler
- Photogate with computer timing system

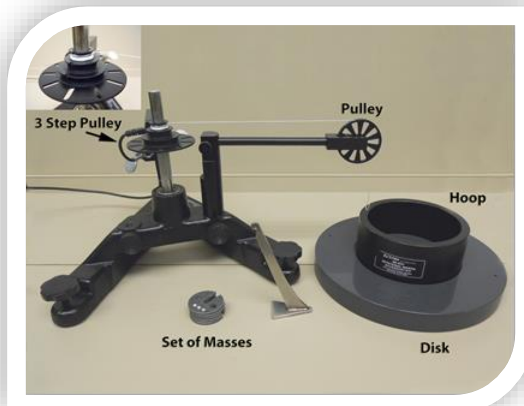


Figure 3

Discussion:

Torque is the tendency of a force to rotate a body about an axis. It depends on the magnitude of the force, the point where the force is applied relative to the axis of rotation, and the direction of the force. It is defined by the vector equation:

$$\left|\vec{\tau}\right| = \left|\vec{R}\right|\left|\vec{F}\right|\sin\theta$$

$\tau = Torque$
$R = radius$
$F = force$
$\theta = angle$ (between force and radius)

This expression can sometimes be simplified by using the effective lever arm. The lever arm represents the distance between the force's line of action and the axis of rotation (suspension point). It is measured on a line that is perpendicular to both (Figure 4). The lever arm's equivalency is given by the following expression:

$$L = \left|\vec{R}\right|\sin\theta$$

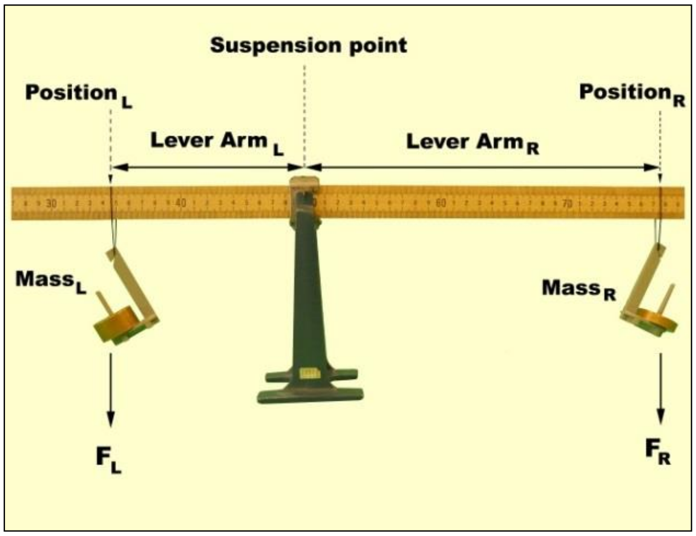


Figure 4

Combining these two expressions, the magnitude of the torque can be calculated using:

$$|\vec{\tau}| = \left|\vec{F}\right|L$$

Where the magnitude of the applied force on this situation (Figure 4) is the force of gravity of the hanging mass:

$$\left|\vec{F}\right| = mg$$

When examining the direction of the torque, it is usually considered positive when the force tends to produce a counterclockwise rotation about the axis, and negative when the force tends to produce a clockwise rotation. If a rigid body is acted upon by a system of torques, where the sum of these torques is zero, the rigid body is in equilibrium with respect to rotation. This means that the body can only have two rotational motion states: to be at rest or to rotate uniformly (constant angular speed) about a fixed axis.

For the cases where the sum of the torques is not zero, the body that has the net torque applied to it is going to experience an angular acceleration. The angular acceleration will be in the same direction as the net torque. This dynamical behavior of rotation has analogous components to the behavior of translational motion where torque (τ) replaces force, angular acceleration ($\boldsymbol{\alpha}$) replaces linear acceleration and moment of inertia ($\boldsymbol{I}$) replaces mass. Again, when the net torque is not equal to zero, the rigid body experiences an angular acceleration in the direction of the net torque. This behavior is described by Newton's second law for rotation:

$$\tau_{net} = I\alpha$$

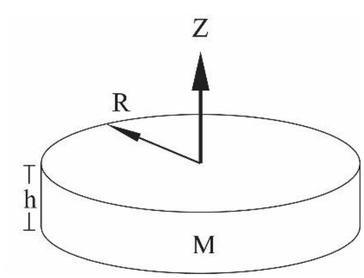
The mass for rotation is replaced by the moment of inertia of the body. It describes not only how much matter a body is composed of, but also how that matter is distributed about the axis of rotation. It is a geometric characteristic of the object, as it depends only on its shape, its mass distribution throughout the shape and the position of the rotation axis. In its discrete form it is defined by:

$$I = \sum_{i=1}^{N} m_i r_i^2$$

This expression means take each particle of mass multiplied by its radius squared (where the radius is the distance from the center of the mass to the center of the axis of rotation) and add them all up to get the total.

$$\boldsymbol{I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 \dots m_N r_N^2}$$

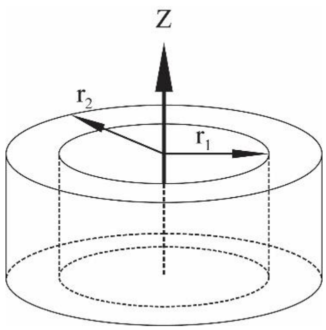
For a disk with radius (*R*) and mass (*M*) rotating about (*z*) axis, the moment of inertia can be calculated by:



$$I_z = \frac{1}{2} M R^2$$

Figure 5

For a thick-walled cylindrical tube with open ends, inner radius ($\boldsymbol{r_1}$), outer radius ($\boldsymbol{r_2}$) and mass (m) rotating about (z) axis, the moment of inertia can be calculated by:



$$I_z = \frac{1}{2}m(r_1^2 + r_2^2)$$

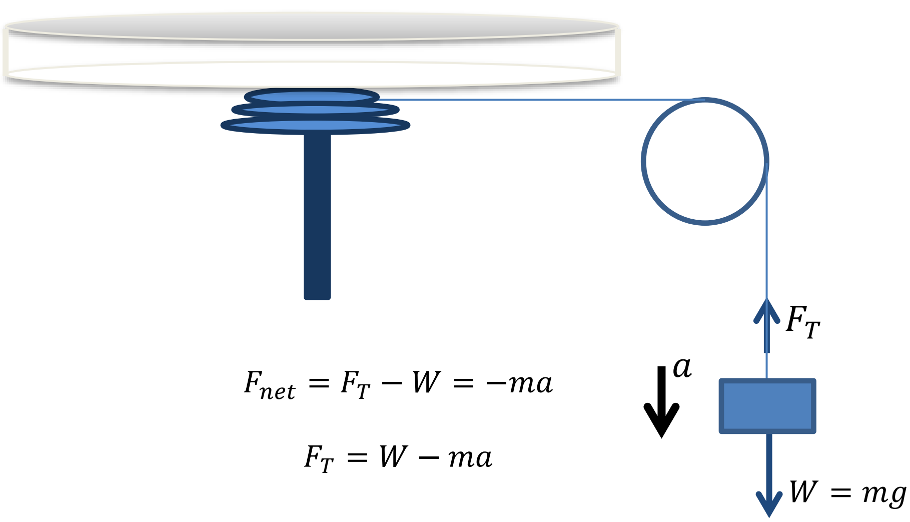
Figure 6

The angular acceleration of the rigid body can be related to the linear acceleration (may be called tangential acceleration) at some point, a radial distance out from the axis of rotation. For cases like this experiment where the system is rotating with a constant angular acceleration (α), it is possible to calculate the linear acceleration (a) for a point located at distance (R) from the center of rotation using the following formula:

$$a = R\alpha$$

The rotation in this experiment is produced by an external applied torque. The force for this torque is from the tension in an attached string ($\boldsymbol{F_T}$) due to a mass hanging on the other end. The weight of the mass is greater than the tension in the string and results in the mass moving with acceleration (a) downward. The magnitude of the tension can be calculated by the following:

$$F_T = m(g - a)$$



$$F_{net} = F_T - W = -ma$$

$$F_T = W - ma$$

Figure 7

For additional information on these concepts please read/review the following sections in your textbook.

Torque & Equilibrium:

	Walker.	**<u>Physics,</u>**	*Chapter* **11** *section* **1** & **3**
	Cutnell & Johnson.	**<u>Physics,</u>**	*Chapter* **9** *section* **1** & **2**

Rotational Dynamics & Moment of Inertia:

	Walker.	**<u>Physics,</u>**	*Chapter* **11** *section* **5**
			Chapter **10** *section* **5**
	Cutnell & Johnson.	**<u>Physics,</u>**	*Chapter* **9** *section* **4**

Procedures:

This experiment consists of two separate parts. The first part is conducted in order to calculate torque and verify rotational equilibrium using the meter stick and the set of masses as shown in Figure 2. The second part is conducted in order to examine rotational dynamics by varying the torque applied to the disk and hoop, determining the resulting acceleration and ultimately calculating the moment of inertia. This part uses the apparatus shown in Figure 3.

Part I: Rotational Equilibrium

1. Record the position of the point of suspension where the meter stick is balanced with no masses attached to it. This represents the center of mass of the meter stick, and can be found by looking at the center, point of suspension, of the clamp. The position should be recorded using units of meters with precision to the millimeter in Table 1.
2. For each trial measure the total mass placed on each side (combined mass and mass hanger) using the most precise balance available. Remember to convert to kilograms.
3. Place a 100-gram mass and hanger on one side of the meter stick by hanging it from the string at some convenient location. Hang a 50-gram mass and hanger on the other side of the meter stick, again from the string but now moving it along the stick until the meter stick reaches equilibrium and is level to the table. Record the position for each mass, using meters, in Table 1.
4. Calculate the lever arm for each mass. This is the distance from the mass to the point of suspension. Next calculate the force and then the torque for each side. Note: the system has reached equilibrium; therefore both values for torque should be identical, or at least very close.
5. For trial 2 place one of the known masses, from the mass kit, on the left side and the unknown mass provided for the right side. ***Note: Make sure the masses differ from those used in trial 1***. Again position them so equilibrium is reached.
6. Complete all the calculations for your known mass, entering the information on Table 2.
7. For the unknown mass, record its position and lever arm. Since we are dealing with a mass of unknown quantity, the torque cannot be directly calculated. But since the system is at equilibrium, we can assume their torques to be identical. Record the same torque value obtained for the known mass as also being the unknown's torque.
8. Use this torque to calculate the force and then the mass of the unknown. Record your results on Table 2.

Part II: Rotational Dynamics

Varying the Radius - Disk

1. The step pulley under the disk has three different radii. Measure each diameter and calculate the radius converting them into units of meters. Record the data in Table 3.
2. Mount the large disk above the step pulley on the center shaft aligning the flat edge of the shaft with the disk.
3. Add 100-grams to a mass hanger and measure the total mass of the combination. The mass will remain constant during this part. Convert the mass to kilograms and record the data in Table 3.
4. Attach the mass hanger to the string. Wind the string on the largest radius until the mass hanger is suspended close to the outside pulley.
5. Adjust the position of external pulley so that the string is aligned with the rod of the pulley.
6. If necessary ask a Lab Assistant to provide computer related instructions to begin the computer data collection and analyses. The computer will display a graph of angular velocity over time. Obtain the statistical slope of the line to find the angular acceleration and its standard deviation for each trial. **Make sure to collect at least five significant digits for the angular acceleration.** Note: when conducting each trial, press start to begin collecting data first and then release the mass. Press stop when the mass hanger reaches the floor.
7. Repeat these steps using the other two radii. Record all of your data in Table 3.

Varying the Force – Disk

8. Place a 75-gram mass on the mass hanger and measure the total mass.
9. The radius will now remain constant during this part of the experiment using only the middle radius. Wind the string until the mass hanger is suspended close to the outside pulley adjusting its position so it is aligned with the mounting rod, as before.
10. Obtain the angular acceleration and its standard deviation as in the above trials. Record the data in Table 3.
11. Conduct one more trials, using 125 grams with the mass hanger. Again obtain the angular acceleration and standard deviation as before. Record your data in Table 3.

Varying the Radius and the Force – Disk and Hoop

12. Next add the hoop on top of the disk being carful center it in the middle of the disk.
13. Repeat the sequence of steps from 2 through 11 as above but now with the hoop added to the top of the disk for each trial. Record all of this data in Table 4.

Theoretical Moments of Inertia

1. Measure the diameter of the disk.
2. Measure the inside and outside diameter of the hoop.
3. Measure the mass of the disk and the hoop separately.

Analysis:

Part II: Rotational Dynamics

1. Calculate the linear acceleration (a), tension (F_T) and torque (τ) for the **first trial** in Table 3. After you have the values calculated for the first trial have a lab instructor confirm the result.
2. The lab instructor will guide you on the remaining calculations for Tables 3 & 4.
3. For the first graph of torque vs angular acceleration use the data on Table 3. Put the torque data on the y-axis and the angular acceleration data on the x-axis. This data should represent a linear relationship. The slope of the statistical linear regression line will be the experimental moment of inertia for the disk.
4. For the second graph of torque vs angular acceleration use the data on Table 4. Put the torque data on the y-axis and the angular acceleration data on the x-axis. Again this data should represent a linear relationship. The slope of the statistical linear regression line will be the experimental moment of inertia for the disk and hoop.
5. Calculate the experimental moment of inertia for the hoop by taking the difference between the two moments, disk and hoop minus disk.
6. Using the dimensions and masses measured for the disk and the hoop; calculate the theoretical moments of inertia for each. Also calculate the percent error for each comparing the experimental value to the theoretical value.

Experiment M25a: Rotational Equilibrium and Rotational Dynamics

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheets: M25a: Rotational Equilibrium and Rotational Dynamics

NAME: ____________________________　　　　**DATE: _________________**

Table 1: Rotational Equilibrium

Location of point of suspension = _________(m)

Trial One					
	mass (kg)	position (m)	lever arm (m)	force (N)	torque (Nm)
left side					
right side					

Table 2: Rotational Equilibrium – Unknown Mass

Unknown mass (recorded from lab balance) = _________ (kg)

Trial Two					
	mass (kg)	position (m)	lever arm (m)	force (N)	torque (Nm)
left side					
	position (m)	lever arm (m)	torque (Nm)	force (N)	calc. mass (kg)
right side					

% Difference between calculated and measured mass = _____________

Data Sheets: M25a: Rotational Equilibrium and Rotational Dynamics

NAME: ____________________________ DATE: _________________

Table 3: Rotational Dynamics - Disk

varying radius	applied mass	radius	angular acc.	stand. dev.	linear acc.	tension	torque

varying mass	applied mass	radius	angular acc.	stand. dev.	linear acc.	tension	torque

Table 4: Rotational Dynamics – Disk and Hoop

varying radius	applied mass	radius	angular acc.	stand. dev.	linear acc.	tension	torque

varying mass	applied mass	radius	angular acc.	stand. dev.	linear acc.	tension	torque

Data Sheets: M25a: Rotational Equilibrium and Rotational Dynamics

NAME: ____________________________ **DATE: _________________**

Experimental Determination Moments of Inertia:

Moment of Inertia for the Disk: __________________

Moment of Inertia for Disk and Hoop: __________________

Moment of Inertia for the Hoop: __________________

Theoretical Determination:

Diameter of the Disk: __________________

Mass of the Disk: __________________

Theoretical Moment of Inertia of the Disk: __________________

Percent Error: __________________%

Inside Diameter of the Hoop: __________________

Outside Diameter of the Hoop: __________________

Mass of the Hoop: __________________

Theoretical Moment of Inertia of the Hoop: __________________

Percent Error: __________________%