M5a: Techniques of Vector Analysis

Introduction:

If someone were to state that they were driving along at a speed of 75 miles per hour in their car when they were stopped and received a ticket for speeding, they have provided their speed as a scalar quantity: a quantity that possessed magnitude (75 mph), but no direction. On the other hand, if that same person bemoaned the fact that they were driving along at a speed of 75 miles per hour traveling West on the East-West Expressway, then they would have provided a vector, a measurement consisting of both magnitude and the direction in which that magnitude is enacted.

The concept of vectors is of vital significance to physics. Without vectors, it becomes impossible to approximate any real life event as represented in more than one dimension. In one dimension, it is possible to make do with only a positive or negative sign to indicate direction, but in any significant representation or approximation of the real world this simplistic system falls short. Instead, in 2 dimensions or more, it is necessary to include both magnitude and direction. The directional component of a 2 dimensional vector is normally determined by setting up an *x*-*y* axis coordinate system, with vectors positioned at the origin of this coordinate system. The direction can be measured by determining the angle in degrees rotating counterclockwise from the + *x* axis.

The purpose of this lab is to provide a clear understanding of exactly how vectors function. Vector addition and subtraction will be performed both graphically through the head-to-tail method and analytically via the component method. The vector results will be demonstrated and confirmed using a vector table, where the length/magnitude of the vectors may be physically approximated via the use of masses. If all calculations are done correctly, when the results are included on the vector table, they will balance out, creating equilibrium.

As a suggestion: because the bulk of this lab is focused on calculations, time spent in the physics lab can be greatly shortened if the vector addition and subtraction is done beforehand.

Apparatus:

- 1 Vector Table
- 1 set assorted masses

** The student is expected to provide metric ruler, protractor, and graph paper.*

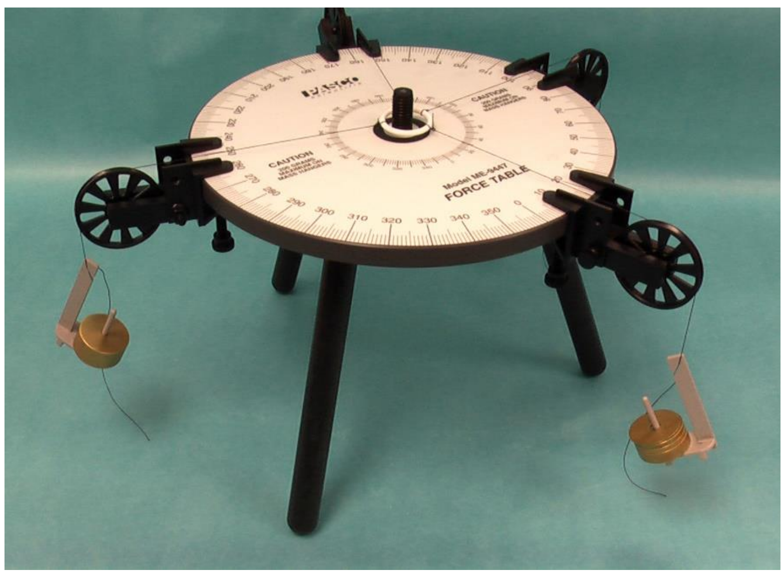

Figure 1

Procedures:

This experiment is divided into three different parts. Both **PART I** and **PART II** require only calculations, and can be done outside of the lab. In both of these first parts, vectors will be added and subtracted graphically and analytically. In **PART III** the results of those calculations will be checked by physically approximating the vectors on a vector table.

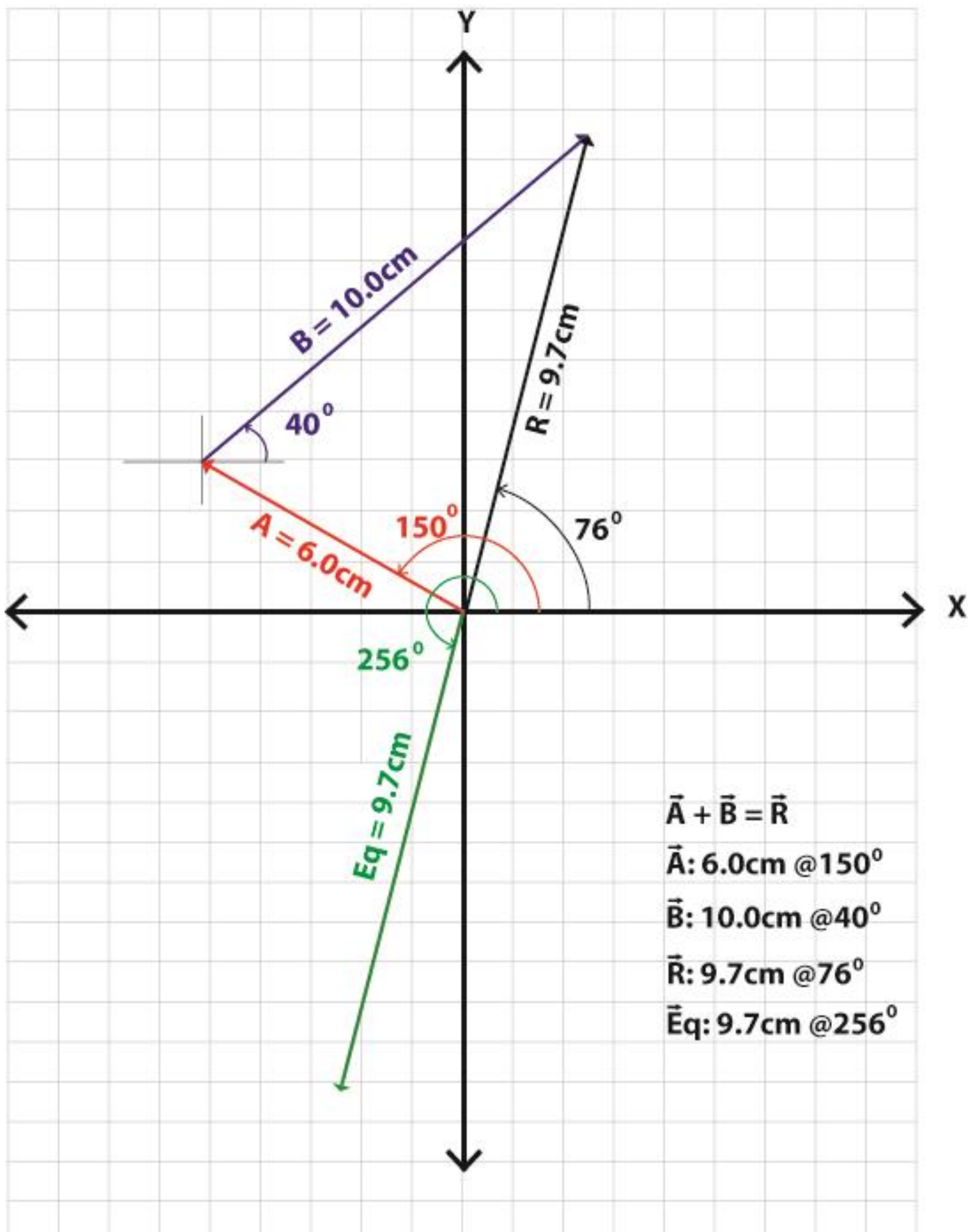

Figure 2. Example of Head-to-Tail Method of Adding Vectors

PART I

1. **Graphically** add the following two vectors using the head-to-tail method. Please check the example given in **Figure 2 a**nd reference any physics textbook available for more information on the head-to-tail method of adding vectors if necessary. It is recommended that the origin for the first coordinate axis be positioned in the middle of the page in order for everything to fit. **Draw the vectors exactly to scale (1:1) and use graphing paper.**

$\vec{A} = 10.0cm @ 50°$

$\vec{B} = 8.0cm @ 160°$ find: $\vec{A} + \vec{B}$

All angles are measured from the +x-axis

2. Find the resultant, $\vec{R}$. Using a protractor and ruler, find its magnitude and direction (from the +x-axis) and label the vector. Record this information on the provided **Data Sheet, Table 1**.
3. The equilibrant is defined as a vector equal in magnitude to the resultant but 180 degrees opposite in direction. The equilibrant is the vector that cancels out the original vector. If one were to use the head-to-tail method to graphically add the equilibrant of $\vec{R}$ to $\vec{R}$, then the resultant vector would be 0. Find the equilibrant to $\vec{R}$ and draw it on the vector diagram. Record its magnitude and angle on the **Data Sheet, Table 1**.
4. **Analytically** determine the magnitude and direction of the resultant, $\vec{R}$. Begin by mathematically finding the *x* and *y* components of $\vec{A}$ and $\vec{B}$. Once again, please reference any physics textbook for more information on finding vector components. Then calculate the sum of all the *x* components and the sum of all the *y* components. Finally, use the sums of the components in conjunction with the Pythagorean Theorem and the arctangent function to determine the resultant. **NOTE: Show your work in a separate page including the equations for the calculations and record all of this information on the provided Data Sheet, Table 2**.
5. **Analytically** determine the magnitude and direction of the equilibrant. Record your results on the **Data Sheet, Table 2**.
6. Find the **percent error** between the graphically determined magnitude of the resultant and the calculated magnitude of the resultant via the vector component method. Repeat these calculations for the angle.

$$\text{percent error} = \left| \frac{x_{\text{experimental}} - x_{\text{theoretical}}}{x_{\text{theoretical}}} \right| * 100\%$$

PART II

1. Repeat the above procedures (steps **1-6** from **PART I**) for the following 3 vectors. Be sure when combining the vectors via the head-to-tail method to make use of a separate sheet of graphing paper from **Part I**'s graph. Record your results on the corresponding **Data Sheet, Tables 3 and 4**.

$\vec{C} = 10.0cm @ 20°$

$\vec{D} = 10.0cm @ 150°$

$\vec{E} = 5.0cm @ 220°$

find: $\vec{C} + \vec{D} - \vec{E}$

All angles are measured from the +x-axis

Part III

1. Now that the resultants and equilibrants for both **PART I** and **PART II** have been found, it is necessary to physically check the results of the experiment using a vector table. First set up the appropriate number of pulleys at the appropriate locations. The pulleys should be aligned at the appropriate angles corresponding to the vectors they approximate. For checking the results from **PART I**, 3 pulleys will be needed ($\vec{A}$, $\vec{B}$, and the *Equilibrant*), and then for confirming **PART II,** 4 pulleys ($\vec{C}$, $\vec{D}$, $-\vec{E}$, and the *Equilibrant*).
2. Once the pulleys have been set up, it will be necessary to approximate the vectors lengths by utilizing masses. For this experiment, let ***10 grams*** approximate ***1.0 centimeter***. Find the mass required for each of the corresponding vectors.
3. Use the light mass hangers provided, as well as the masses in the mass kit to compile the required mass for each vector. If a mass included in the mass kit is not labeled, it is possible to check its mass on any of the laboratory's balances (rounding the measured mass to the nearest whole number). Please keep in mind that the mass hangers themselves will factor into the total mass hanging from the strings, and must be taken into account.
4. Once the appropriate masses have been assembled, connect the mass hangers and their masses to the strings connected to the ring on the vector table by wrapping the string around the mass hangers several times. Do not tie the strings to the mass hangers.
5. Physically check your resultant by using the appropriate equilibrant. Place the corresponding mass for the resultant at the angle you previously determined for the equilibrant. The system should remain in equilibrium (balanced and centered around the vector table pin in the center of the vector table). Take notes or pictures to describe and explain the equilibrium state of your vector table for the lab write up. **Please make sure to have a lab instructor confirm that the system is balanced for both PART I and PART II.**

Do not draw PART I and PART II on the same page. Also, be sure to include the equilibrant for each graph on the same page as the vectors were graphically drawn.

Experiment M5a: Techniques of Vector Analysis

Student Name_______________________________________

*Lab Partner Name*_______________________________________

*Lab Partner Name*_______________________________________

Physics Course_______________________________________

Physics Professor_____________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Result Summary: M5a: Techniques of Vector Analysis

NAME: ____________________________ **DATE: _________________**

PART I: $\vec{R} = \vec{A} + \vec{B}$

Table 1

Graphical (Head-to-tail) Method of Vector Addition	
Magnitude of $\vec{R}$ (measured with ruler):	
Angle of $\vec{R}$ (measured with protractor):	
Magnitude of the Equilibrant (measured with ruler):	
Angle of the Equilibrant (measured with protractor):	

Table 2

Component Method of Vector Addition			
A_x		A_y	
B_x		B_y	
R_x		R_y	
Magnitude of $\vec{R}$ (calculated):			
Angle of $\vec{R}$ (calculated):			
Magnitude of the Equilibrant (calculated):			
Angle of the Equilibrant (calculated):			

% Error for Magnitude of $\vec{R}$
% Error for Angle of $\vec{R}$

Show Calculations for Component Method of Vector Addition below:

PART II: $\vec{R} = \vec{C} + \vec{D} - \vec{E}$

Table 3

Graphical (Head-to-tail) Method of Vector Addition	
Magnitude of $\vec{R}$ (measured with ruler):	
Angle of $\vec{R}$ (measured with protractor):	
Magnitude of the Equilibrant (measured with ruler):	
Angle of the Equilibrant (measured with protractor):	

Table 4

Component Method of Vector Addition			
C_x		C_y	
D_x		D_y	
E_x		E_y	
R_x		R_y	
Magnitude of $\vec{R}$ (calculated):			
Angle of $\vec{R}$ (calculated):			
Magnitude of the Equilibrant (calculated):			
Angle of the Equilibrant (calculated):			

% Error for Magnitude of $\vec{R}$
% Error for Angle of $\vec{R}$

Show Calculations for Component Method of Vector Addition below: