Experiment O17a. Thin Lenses.

Introduction:

Lenses are crucial parts of many optical instruments, starting from a simple magnifying glass, to modern optical telescopes. They also play an important role in everyday life such as the correction of nearsightedness and farsightedness.

When a ray of light passes through a lens, it changes its direction of propagation due to refraction at the boundaries between air and lens. Refraction is responsible for formation of an image – a reproduction of an object via light.

In this experiment you will investigate characteristics of images produced by a single converging lens and by a combination of a converging and a diverging lens. You will also determine the focal length~~s~~ of a converging lens.

Apparatus:

- Laser & laser holder
- Two meter optical rail
- 2 fixed filter mounts
- 6 Optical rail carriers, optical post holders, and optical posts
- 2 converging double convex (DCX) lenses (lens 1 and lens 2) with two holders
- A converging and a diverging lenses (lenses 3 and 4) of unknown focal length
- Crossed arrow target object and diffuser

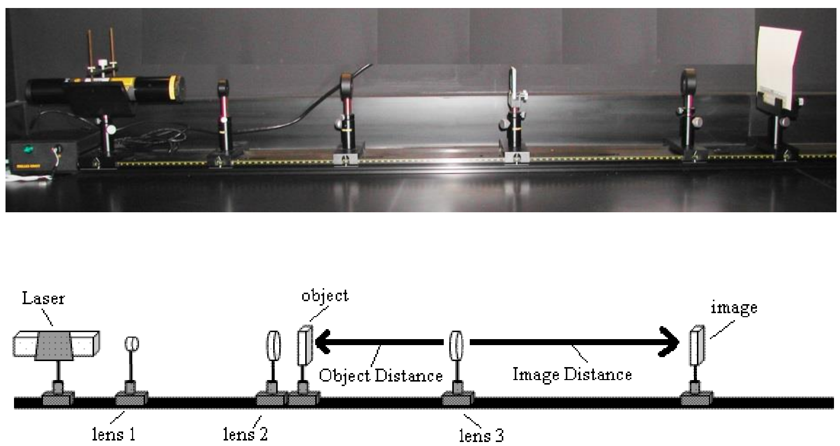

Figure 1. Experimental setup for Part I: investigating images produced by a converging lens (lens 3).

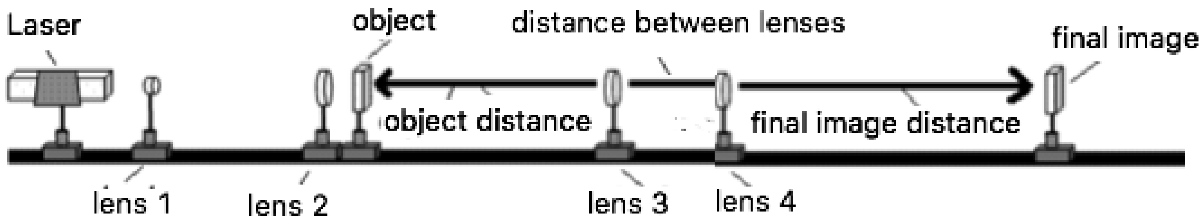

Figure 2. Experimental setup for Part II: investigating images produced by a combination of two lenses (lens 3 and lens 4).

The optical rail system shown in Figures 1 and 2 has many optical instruments mounted along its two meter length. Most of the instruments, like lenses 1 & 2, will remain stationary throughout the experiment. These lenses, together with the laser, are used to illuminate the object.

Theoretical Background:

Images formed by a lens or a combination of lenses can be real or virtual. The image is called real if it can be projected on a screen (exists whether or not an observer is present). The image is called virtual, if it cannot be projected on a screen, but appears to an observer. An example of a virtual image is an enlarged image of an object one can see through a magnifying glass.

Spherical lenses are made from a transparent material, polished in the form of segments of two spheres. There are two types of spherical lenses: converging and diverging. When parallel rays of light pass through a converging lens, due to refraction they "converge" toward one point, called the focal point. Distance between the focal point and the lens is called the focal length of the lens. When parallel rays of light pass through a diverging lens, they "diverge" and appear to emanate from one point, called the focal point of a diverging lens. The distance between this point and the lens is called the focal length of a diverging lens. The focal length depends on the index of refraction of the material the lens is made of, shape (concave, convex, or planar) of its surfaces, and its radii of curvature.

For both converging and diverging lenses, the relationship between object distance and image distance is given by *the thin lens equation*:

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}, \qquad (1)$$

where f is the focal length of the lens, d_o is the object distance, and d_i is the image distance.

You can find the derivation of the thin lens equation in your textbook.

The focal length of a converging lens is always positive. Images generated by a single converging lens are *real* if the object distance is larger than the focal length of the lens; images are *virtual* if object distance is less than focal length of the lens. The focal length of a diverging lens is always negative (it diverges light). Images generated by a single diverging lens are always virtual.

To locate a real image, one can move a screen to the position where the image appears the sharpest. Found this way, distance between the screen and the lens represents distance to the image (d_i). For a real image generated by a single converging lens, both object distance and image distance are always positive.

For an image generated by a single lens, the *lateral magnification*, *m*, is defined as a ratio of the height (size) of an image, h_i, to the height of the object, h_o:

$$m = -\frac{h_i}{h_o}. \qquad (2)$$

The minus sign in equation (2) indicates that a real image generated by a single converging lens is inverted (upside down) with respect to the object. Using properties of similar triangles, it can be shown that the lateral magnification can also be calculated using the ratio of image distance to object distance,

$$m = -\frac{d_i}{d_o}. \qquad (3)$$

When parallel rays of light pass through a combination of two lenses, they experience changes in direction of propagation twice: once due to the two surfaces of the first lens (the lens which is closer to the object), and again due to the two surfaces of the second lens. Using geometry, it can be shown that to calculate the image location produced by a combination, one needs to use the thin lens equation (1) twice. First, applying it to the first lens and finding the location of a “primary image”. The primary image serves as an “object” for the second lens. Applying the thin lens equation for the second lens allows one to find the location of the final image.

For the first lens, object distance, d_{o1}, is the distance from the object to the first lens. This distance is *always* positive. For the second lens, object distance, d_{o2}, is the distance from the primary image, generated by the first lens, to the second lens. This distance can be positive *or* negative, depending on location of the primary image. If the primary image is located at the same side from the second lens as original object, d_{o2} is positive. If the primary image is located on the side of the second lens *opposite* the original object, d_{o2} is negative. In this case the primary image serves as a “virtual object” for the second lens.

The overall lateral magnification for a combination of lenses, M, is a product of magnifications by both lenses:

$$M = -\frac{h_{i2}}{h_o} = m_1 \times m_2 = \left(-\frac{d_{i1}}{d_{o1}}\right) \times \left(-\frac{d_{i2}}{d_{o2}}\right), \quad (4)$$

where h_{i2} is the size of the final image, h_o is the size of the object, d_{i1} is the primary image distance, d_{o1} is the first object distance, d_{i2} is final image distance, and d_{o2} is the second object distance.

Determining the position of the final image produced by a combination of lenses and overall magnification is not hard, but can be confusing in the beginning. To help you, four examples are provided in the Appendix to this lab handout. You are also highly recommended to read the appropriate portions of your chapter 34 in your textbook.

Experimental Procedure:

Before beginning, please see the laboratory personnel for instructions concerning the proper handling and care of the optical equipment. Always wash and dry hands before interacting with optics equipment. Do not attempt to start this lab without checking with an instructor.

Part I: Images produced by a converging lens and the focal length of a converging lens.

In the first part of the experiment you will vary the object distance to a converging lens, locate an image, determine image distance and measure the size of an image. You will use equation (1) to determine the focal length of the lens and equations (2) and (3) to compare measured size of an image with the expected size.

After having washed your hands, verify that the equipment is set up as shown in Figure 1. Also, be sure to check the positions of the laser, lens 1, lens 2 and the object, making sure that the positions are the same as those specified in Table 1 (posted on the wall above the equipment). *It is not necessary to touch the laser, lens 1, lens 2, or the object at any point during the lab,* unless the equipment is not set up to the specifications of Table 1. If needed, re-assemble the equipment so that it matches the diagram. Then ask a lab assistant to confirm that the setup is correct.

1. Obtain a converging lens from a lab instructor and insert it in a holder. This will be your "lens 3" in this exercise. Record its actual focal length, as provided by the lab instructor, in Table 1.
2. Choose six positions for lens 3. The choices should be separated by at least 2.00 cm from each other on the bench, and should allow for the formation of a real image. Note: a converging lens will produce a real image of an object if the distance to the object is

larger than the focal length of the lens. Record your chosen positions in Data Table 1 and show them to a lab instructor.

3. Move lens 3 to the first position you wrote down in Data Table 1.
4. Slowly move the image screen along the optical rail until the image of the target appears as sharply focused as possible. The position of the screen found this way is the image position.
5. Record the image position in the Data Table 1.
6. Use a caliper to estimate the size of the target circle's *outside* diameter. This will be an approximate measurement. Record this measurement as the image size in the Data Table 1.
7. Repeat these steps for each lens 3 position written in Data Table 1.

Analysis:

8. Determine distances to the object and the image, and calculate the focal length of lens 3 using equation (1). Record your calculated values in Analysis Table 1.
9. Calculate the mean and standard deviation of the focal lengths you determined in previous step. Compare your experimentally determined focal length with the actual value provided by the instructor.
10. Calculate the magnification factor and theoretical image size for each image position. Record your obtained values in Analysis Table 1. Help with analysis is provided in the Appendix.
11. Calculate the percent error for each measured image size.

Part II. Images produced by a combination of lenses.

A. A diverging lens in front of a converging lens.

1. Obtain a diverging lens from the instructor, record its focal length. This will be your "lens 4".
2. For **first trial**, place a holder for lens 4 between lens 3 and the object, at 5-10 cm from the object.
3. Position lens 3 at a distance from lens 4 which is *larger* than the focal length of lens 3.
4. Move the image screen to sharpen the image. Record the image position.
5. Measure and record the size of the image. Notice and record the orientation of the image with respect to the object (upright or inverted).
6. Draw a neat diagram of the setup, including the object, lens 3, lens 4, and the screen. Specify the precise positions of each of the elements on your diagram.
7. For **second trial**, slowly move lens 3 by a few cm, keeping the distance between lens 3 and lens 4 *larger* than the focal length of lens 3. Repeat steps 4-6.
8. For both trials, determine distance between the experimentally found final image and first lens (diverging lens 4).
9. For both trials, following Example 1, calculate the theoretical distance between lens 3 and final image, d_{o2}, and the overall magnification. Then calculate the theoretical distance between lens 4 and the final image.

10. Calculate the percent error between your theoretical and experimentally determined distances (between the first lens in the combination and the final image). Also calculate the percent error in the overall magnification.

B. A diverging lens behind a converging lens.

1. Carefully remove lens 4 (with the holder) from the bench.
2. For **first trial**, slowly move lens 3 to one of positions specified in Data Table 1.
3. Slowly move the screen to locate the image. This image is a "primary image" for a combination of lenses. Record the position of this primary image again.
4. Move the screen farther from lens 3 and carefully place lens 4 back on the bench between lens 3 and the screen. Lens 4 must be a few centimeters *closer* to lens 3 than the position of the image you located in previous step, and the distance from lens 4 to the primary image must be less than the absolute value of the focal length of lens 4.
5. Move the screen to sharpen the final image. If you have a difficulty doing this, ask an instructor for help. Record the final image position.
6. Measure and record the size of the final image. Record the orientation of the final image with respect to the object (upright or inverted).
7. Draw a clear diagram of the setup, including the object, lens 3, lens 4, and the screen. Specify the precise positions of each of the elements on your diagram.
8. For **second trial**, slowly move lens 3 to a different position specified in Table 1. Carefully remove lens 4 from the bench again and repeat steps 3-7.
9. For both trials, determine distance between the experimentally found final image and first lens (converging lens 3).
10. For both trials, following Example 1, calculate the theoretical distance between lens 4 and final image, d_{o2}, and the overall magnification. Then calculate the theoretical distance between lens 3 and the final image.
11. Calculate the percent error between your theoretical and experimentally determined distances (between the first lens in the combination and the final image). Also calculate the percent error in the overall magnification.

Table 1.

Optical Component	**Position on Optical Rail (mm)**
Laser	90.0
lens 1 f = +15.60 mm	235.0
lens 2 f = +400.0 mm	650.0

object (target diameter = 10.0 mm)	700.0
lens 3 *f* = _________________	
lens 4 *f* = _________________	

Experiment O17a: Thin Lenses.

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheets: O17a: Thin Lenses.

NAME: ______________________________ DATE: __________________

Laser position: _____________________

Lens 1 position: ____________________ ***f*** = _____________

Lens 2 position: ____________________ ***f*** = _____________

Lens 3 focal length: - - - - - - - - - - - - - ***f*** = _____________

Object size: _______________________

Part I.

Data Table 1.

Position (in mm)			Size (in mm)
Object	Lens 3	Image	Image (approx.)
Finding Image Positions:			

Analysis Sheets: O17a: Thin Lenses.

NAME: ____________________________ DATE: _________________

Part I.

Analysis Table 1. Determining the focal length of a converging lens.

Distance (in mm)		Focal length	Magnification Factor	Image Size (in mm) (calculated)	% error
Object	Image				(Image size)
Finding Image Positions:					

Mean focal length___________________________________

Standard deviation__________________________________

Data Sheets: O17a: Thin Lenses.

NAME: ____________________________ **DATE: _________________**

Part II A.

Lens 3 focal length: - - - - - - - - - - - - ***f*** **=** _____________
Lens 4 focal length: - - - - - - - - - - - - ***f*** **=** _____________

First trial.
Object position______________________________
Lens 3 position: _____________________________
Lens 4 position: _____________________________
Final image position: _________________________
Image size and orientation: ____________________

Diagram:

Second trial.
Object position______________________________
Lens 3 position: _____________________________
Lens 4 position: _____________________________
Final image position: _________________________
Image size and orientation: ____________________

Diagram:

Analysis Sheets: O17a: Thin Lenses.

NAME: ______________________________ DATE: __________________

Part II A Calculations:

<u>First trial.</u>

Lens 3 (2nd lens in combination): focal length:________ position: ________ Lens 4: (1st lens in combination) focal length:________ position: ________	Calculations (show equations and work)	Calculated value
d_{o1}		
d_{i1}		
d_{o2}		
d_{i2}		
d (Distance between first lens in combination and final image) (Theoretical)		
d (Distance between first lens in combination and final image) (Experimental)		
% error for d		
Magnification factor (Theoretical)		
Magnification factor (Experimental)		
% error for Magnification Factor		

Part II A Calculations:

<u>Second trial.</u>

Lens 3 (2nd lens in combination): focal length:_______ position: _______ Lens 4 (1st lens in combination): focal length:_______ position: _______	Calculations (show equations and work)	Calculated value
d_{o1}		
d_{i1}		
d_{o2}		
d_{i2}		
d (Distance between first lens in combination and final image) (Theoretical)		
d (Distance between first lens in combination and final image) (Experimental)		
% error for d		
m (Theoretical)		
m (Experimental)		
% error for Magnification Factor		

Data Sheets: O17a: Thin Lenses.

NAME: ______________________________ **DATE: __________________**

Part II B.

Lens 3 focal length: - - - - - - - - - - - - - ***f*** = ______________
Lens 4 focal length: - - - - - - - - - - - - - ***f*** = ______________

First trial.
Object position________________________________
Lens 3 position: _______________________________
Primary image position: ________________________
Lens 4 position: _______________________________
Final image position: ___________________________
Image size and orientation: _____________________

Diagram:

Second trial.
Object position________________________________
Lens 3 position: _______________________________
Primary image position: ________________________
Lens 4 position: _______________________________
Final image position: ___________________________
Image size and orientation: _____________________

Diagram:

Part II B Calculations:

<u>First trial.</u>

Lens 3 (1st lens in combination): focal length:________ position: ________ Lens 4 (2nd lens in combination): focal length:________ position: ________	Calculations (show equations and work)	Calculated value
d_{o1}		
d_{i1}		
d_{o2}		
d_{i2}		
d (Distance between first lens in combination and final image) (Theoretical)		
d (Distance between first lens in combination and final image) (Experimental)		
% error for d		
m (Theoretical)		
m (Experimental)		
% error for Magnification Factor		

Part II B Calculations:

<u>Second trial.</u>

Lens 3 (1st lens in combination): focal length:_______ position: _______ Lens 4 (2nd lens in combination): focal length:_______ position: _______	Calculations (show equations and work)	Calculated value
d_{o1}		
d_{i1}		
d_{o2}		
d_{i2}		
d (Distance between first lens in combination and final image) (Theoretical)		
d (Distance between first lens in combination and final image) (Experimental)		
% error for d		
m (Theoretical)		
m (Experimental)		
% error for Magnification Factor		

Appendix.

I. Help with Analysis for Part I.

First, it is necessary to calculate the object distance and image distance for *each* lens 3 position. Calculate the object distance and the image distance using the positions of the object, the image, and lens 3 recorded in the Data Table 1. The object distance and image distance should then be entered into Analysis Table 1.

From these positions (object distance and image distance), it is possible to calculate the focal length for lens 3, using the thin lens equation. Then calculate the magnitude of the magnification factor via the formula for lateral magnification, using image distance and object distance. After calculating the magnification factor, calculate the theoretical image size by using the lateral magnification factor and the object (target) size. Compare the theoretical and measured values.

II. Possible scenarios you may encounter while attempting to investigate an image generated by a *combination* of a converging and a diverging lenses.

Example 1: Virtual primary image, real final image.

A 3.0 cm high object is located at 6.0 cm in front of a diverging lens of focal length $f_1 = -4.00$ cm. A converging lens of focal length $f_2 = 8.00$ cm is located 10.0 cm behind the diverging lens. Determine the location of the final image and the overall magnification.

Solution:

First, let's find the location of the primary image. The distance from the object to the first lens is $d_{o1} = 6.0$ cm. Applying equation (1) to the first lens yields $d_{i1} = -2.4$ cm. The primary image is virtual and 2.4 cm in front of the diverging lens, on the same side as the object.

Second, let's determine the distance to the primary image to the second lens. Because the second lens is 10.0 cm behind the first lens, this distance is 2.4+10.0 = 12.4 cm. The primary image is located on the same side of the second lens as original object, thus the distance to object 2 is positive: $d_{o2} = 12.4$ cm.

Applying the thin lens equation to the converging lens yield $d_{i2} = 22.5$ cm. the distance to image 2 is positive, therefore the final image is *real* and can be projected on a screen.

The overall magnification $M=(-d_{i1}/d_{o1}) \times (-d_{i2}/d_{o2})=(-(-2.4)/6.0) \times (-22.5/12.4) = -0.726$

Example 2: Virtual primary image, virtual final image.

A 3.00 cm high object is located 5.00 cm in front of a diverging lens of focal length $f_1 = -4.00$ cm. A converging lens of focal length $f_2 = 8.00$ cm is located 5.00 cm *behind* the diverging lens. Determine the location of the final image and the overall magnification.

Solution:

The distance from the object to the first lens is d_{o1} = 5.00 cm. Applying equation (1) to the first lens yields d_{i1} = -2.22 cm. The primary image is *virtual* and 2.22 cm in front of the diverging lens, the same side of the lens as the object.

Because the second lens is 5.00 cm behind the first lens, the distance to object 2 is

2.22 + 5.00 = 7.22 cm. The primary image is located on the same side of the second lens as the original object and the distance to object 2 is positive: d_{o2} = 7.22 cm.

Applying the thin lens equation to the converging lens yield d_{i2} = -74.1 cm. The distance to image 2 is negative, therefore the final image is *virtual* and cannot be projected on a screen.

The overall magnification $M = (-d_{i1}/d_{o1}) \times (-d_{i2}/d_{o2}) = (-(-2.22)/5) \times (-(-74.1/7.22)$ = 4.55 cm. Therefore, the final image has the same orientation as the object.

Example 3: Real primary image, real final image.

A 3.00 cm high object is located at 10.0 cm in front of a converging lens of focal length f_1=8.00 cm. A diverging lens of focal length f_2=-4.00 cm is located 37.0 cm behind the converging lens. Determine the location of the final image and the overall magnification.

Solution:

First, let's find the location of the primary image. The distance to the object for the first lens is d_{o1} = 10.0 cm. Applying equation (1) to the first lens yields d_{i1} = 40.0 cm. Therefore, the *primary image* is 40.0 cm behind the converging lens.

Next, let's determine distance from the primary image to the second lens. Because the second lens is 37.0 cm behind the first lens, this distance is 40.0 – 37.0 = 3.0 cm. However, because the primary image is located on the *opposite* side of the second lens, the distance to object 2 is

d_{o2} = -3.0 cm.

Applying the thin lens equation to the diverging lens yields d_{i2} = 12.0 The distance to image 2 is positive, therefore the final image is *real* and can be projected on a screen.

The overall magnification can be found using equation (4):

$M = (-d_{i1}/d_{o1}) \times (-d_{i2}/d_{o2}) = (-40/10) \times (-12.0/-3.0) = -16.0$. The negative sign in front of magnification indicates that the final image is inverted with respect to the object.

Example 4: Real primary image, virtual final image.

A 3.00 cm high object is located at 16.0 cm in front of a converging lens of focal length f_1=8.00 cm. A diverging lens of focal length f_2 = -4.00 cm is located 20.0 cm behind the converging lens. Determine the location of the final image and the overall magnification.

Solution:

Now, the distance from the object to the first lens is d_{o1} = 16.0 cm. Applying equation (1) to the first lens yields d_{i1} = 16.0 cm. Therefore, the primary image is 16.0 cm behind the converging lens.

Because the second lens is 20.0 cm behind the first lens, this distance is 20.0-16.0 = 4.00 cm. The primary image is located on the *same* side of the second lens as the original object and the distance to object 2 is positive: d_{o2} = 4.00 cm.

Applying the thin lens equation to the diverging lens yield d_{i2} = -2.00cm. The distance to image 2 is negative, it is located between the two lenses therefore the final image is *virtual* and cannot be projected on a screen.

One can still find the overall magnification using equation (4):

$M = (-d_{i1}/d_{o1}) \times (-d_{i2}/d_{o2}) = (-16/16) \times (-(-2)/4) = -0.500$. Thus, the final image is virtual *and* inverted with respect to the object.