O2a: Determining the Index of Refraction of Light

Introduction:

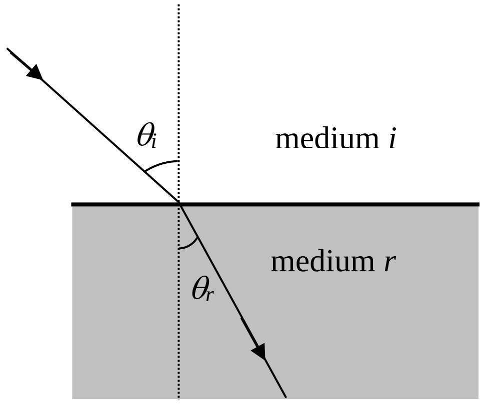


Figure 1

Light travels through the vacuum of empty space at the ultimate speed limit of the universe. The speed of light can change, though, depending upon the media light travels through. Light slows down differently each time it passes through other media (such as water or glass).

This difference in speed as light transitions from one medium to another causes a "bending" in a ray of light if the ray entering the different medium is at an angle other than 90°. This shifting, known as refraction of light, and the amount of angular deviation that the light ray refracts are both related to a dimensionless quantity called the index of refraction. This relationship is expressed via Snell's law, formulated by Dutch mathematician William Snellius in 1621 (though several others also discovered the law, including Rene Descartes in 1637):

Snell's Law of Refraction: $n_i \sin\theta_i = n_r \sin\theta_r$

(n is the index of refraction for a given medium)

The index of refraction, n, is defined by the ratio of the light's speed in a vacuum to the light's speed in the medium.

$$n \equiv \frac{c}{v}$$

Where: *c = speed of light in a vacuum*
v = speed of light in the medium

The purpose of this lab is to determine the index of refraction for a piece of clear glass. The index of refraction will be found by measuring the displacement caused in the laser light as the glass is rotated to different angles. From the original angles of incidence θ_i and the distance the laser light is displaced, it is possible to determine the angles of refraction θ_r using geometry. With these two angles, it is a simple process to determine the index of refraction by using Snell's Law.

Apparatus:

- Laser
- Optical rail
- Component carriers
- Mounting hardware
- Angular translator
- Linear translator
- Photometer
- Glass plate
- Comp. w/ interface

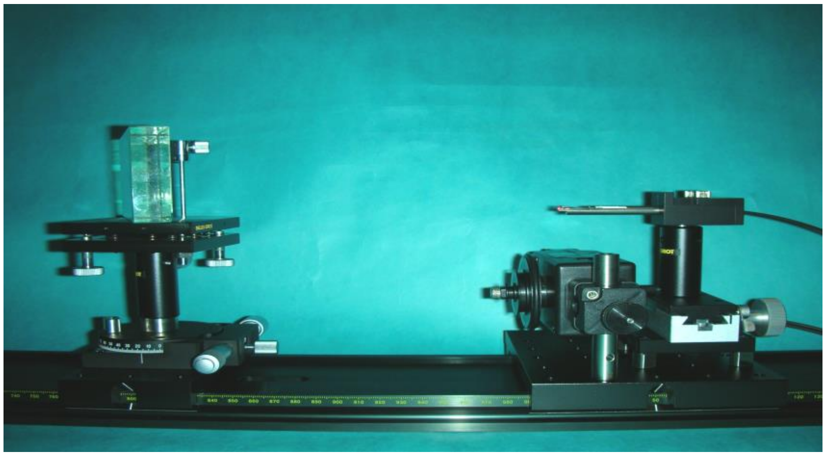


Figure 2

Discussion:

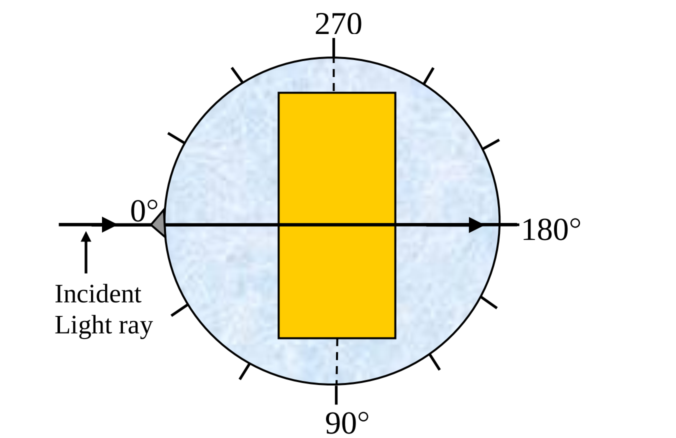


Figure 3

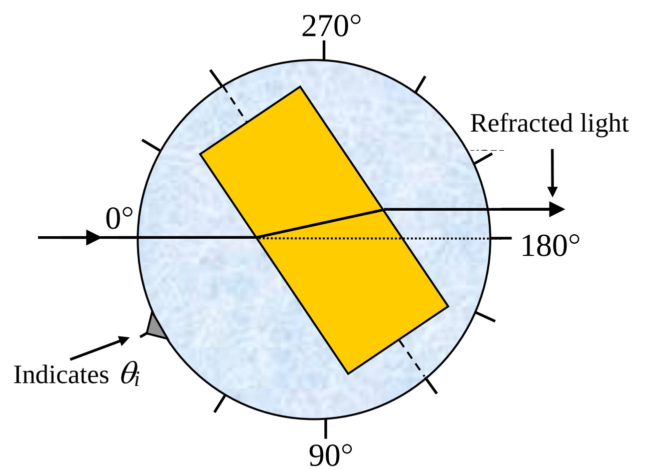


Figure 4

The material for this experiment is a thick glass plate approximately (6 x 11 x 2) centimeters. The glass will first be positioned on an angular translator, perpendicular (at 90°) to the incoming ray of light, where no refraction of the light beam occurs. Then the material will be rotated to several different angles (with the angle determined by the setting of the angular translator), causing light to be refracted within the material.

A linear translator, fitted with the eye of a photometer that can sense the intensity of the laser light, is set at the opposite end of the optical rail, and can carefully and slowly be moved in order to measure the position of the light ray as it changes for each of the different angles. The displacements of the ray of light, along with the thickness of the material, can then be used to calculate the angle of refraction, θ_r, for each incidence angle.

A top view representation of the light traveling through the glass while sitting on the angular translator has been provided in figures three and four.

From this representation in **Figure 4**, which has been expanded into **Figure 5**, in order to show it more clearly, it becomes possible to derive the equation necessary to find θ_r. Making use of the geometric fact that two intersecting lines form vertical angles which are congruent, it can be seen that for **Figure 5**: $\theta_i = \theta_r + \theta_\beta$. From this equation, it is not at all difficult to derive the following relationship that will be needed as θ_r is found: $\theta_\beta = \theta_i - \theta_r$

Also apparent from closer consideration of **Figure 5** is the fact that the dotted line through the glass block is of length ***t***, where ***t*** represents the thickness of the block. Additionally, it can be seen that the shifting of the laser light has created two right triangles: the first has an angle of θ_r, an adjacent side with length ***t***, and a shared hypotenuse that has been labeled ***x***. The second triangle has an angle of θ_β, an opposite side of length ***d*,** where ***d*** is equal to the displacement of the laser light, and a shared hypotenuse ***x***.

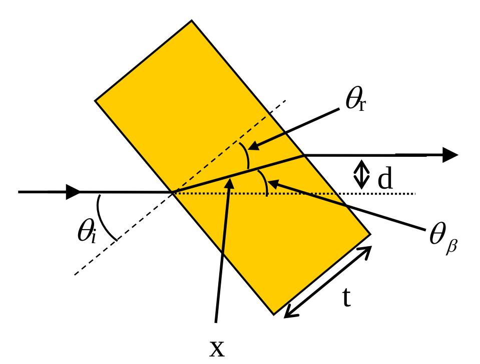


Figure 5

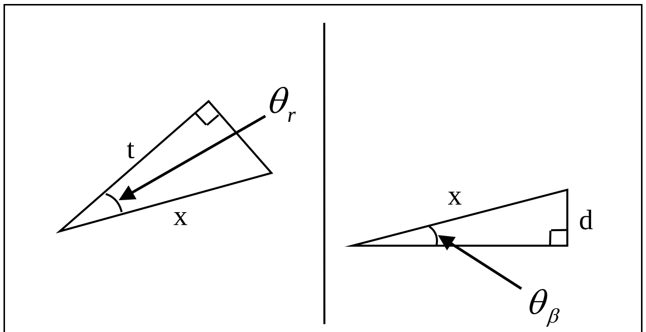


Figure 6

The layout of the two right triangles created by the laser light's displacement and shown in **Figure 5** can be seen much more clearly if one separates those two triangles out, as done in **Figure 6**.

From the provided picture and geometrical definitions, the following relationships may be stated:

$$\cos\theta_r = \frac{t}{x} \qquad \sin\theta_\beta = \frac{d}{x}$$

Now, by solving both equations so that x is isolated, and then setting the two equations equal to each other, the following result may be obtained:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin\theta_\beta}.$$

From here, one is able to make use of the fact that $\theta_\beta = \theta_i - \theta_r$, in order to find the following equation in terms of θ_i and θ_r:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin(\theta_i - \theta_r)}$$

At this point, it becomes necessary to consider the following trigonometric identity**:** $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$. By applying this identity, the above equation is transformed to yield up:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin\theta_i\cos\theta_r - \cos\theta_i\sin\theta_r}$$

Now, by cross multiplying, one can obtain that:

$$\frac{\sin\theta_i\cos\theta_r - \cos\theta_i\sin\theta_r}{\cos\theta_r} = \frac{d}{t}$$

And by then simplifying this equation, it becomes:

$$\sin\theta_i - \cos\theta_i\tan\theta_r = \frac{d}{t}$$

From here, one simply has to subtract the $\frac{d}{t}$ over to one side, add the $\cos\theta_i\tan\theta_r$ to the other side and divide out the $\cos\theta_i$ to isolate θ_r.

$$\frac{\sin\theta_i - \frac{d}{t}}{\cos\theta_i} = \tan\theta_r$$

This, in turn, yields up the needed equation to calculate the angle of refraction θ_r.

Angle of Refraction: $\theta_r = \tan^{-1}\left(\frac{\sin\theta_i - (d/t)}{\cos\theta_i}\right)$ Where:

d = displacement
t = thickness of material
θ_i = angle of incidence
θ_r = angle of refraction

After obtaining the angle of refraction use it together with the angle of incidence to calculate the index of refraction by using Snell's Law.

Snell's law: $$\mu = \frac{\sin\theta_i}{\sin\theta_r}$$

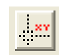
Note: The index of refraction's notation has been changed from n to μ because for this experiment μ is the relative index of refraction (relative to air): $\mu = \frac{n_r}{n_i}$, where $n_i \approx 1$ for air.

Procedures:

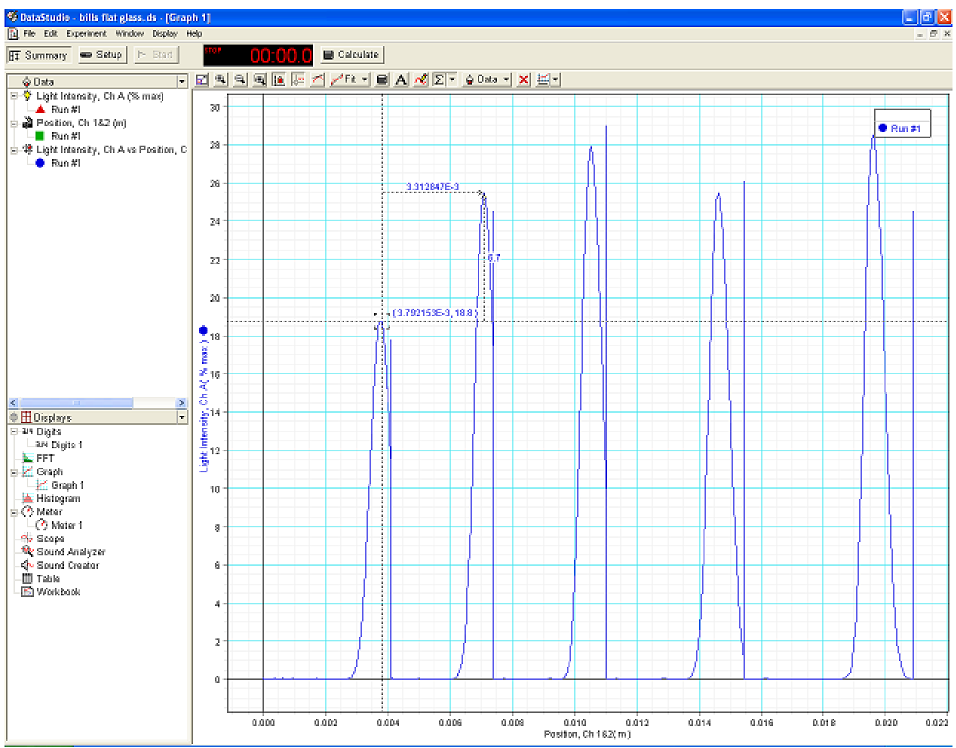
1. Measure the thickness of the glass plate. The plate will be laid on its side so this will correspond to the width of the plate.
2. Position the angular translator to point at zero degrees.
3. Mount the glass plate on the angular translator by laying it down on the platform attached above the angular translator. Adjust the plate's position such that it is perpendicular to the laser beam. When the position is correct the light being reflected from front surface of the glass is directed exactly back into the laser. Be carful to not change the angle of the angular translator, it should remain on zero while making this adjustment.
4. Open data studio to the correct program being used for this experiment.
5. Confirm the angular translator position is 0 degrees and adjust the linear translator position to be offset right of the laser beam ray. Start the computer data collection.
6. Begin collecting data by ***slowly and smoothly*** turning the knob of the sensor until you get a complete maximum peak (which will look like a tall hump) on the graph.
7. Now pause moving the linear translator but do not stop the computer collecting data.
8. Carefully position the angular translator to 10 degrees and repeat process (steps 6 & 7).
9. Continue each trial by increasing the incidence angle by 5 degrees and repeating the process (steps 6 & 7) until you finish with 55 degrees. You should have 11 peaks on the graph after completing the angles through 55 degrees.

Analysis:

Once you have collected your data on the graph you need to obtain the light ray's displacement from its original position due to refraction as it travels through the glass. The displacement is the difference in the position from the maximum peak at zero degrees to the maximum peak at each incidence angle.

First click on the xy "smart tool"  in the upper part of the graph.

Position the cross hair on the middle top of the first peak, corresponding to the zero degree maximum. This will be your zero degree position. It may be necessary to zoom into the region to precisely adjust the cross hair to the maximum. The coordinates are in (x,y) form and will be written along the horizontal line. The position corresponds to the x-axis, therefore record the first coordinate as the zero degree position. Now move the cross hair over to the next peak corresponding to the 10 degree maximum. Again zoom into the region as necessary to obtain a precise measurement of the maximum position. Record this as the 10 degree position. Next move the cross hair from the 10 degree peak to the 15 degree peak and record this next position. Continue these same steps, moving the cross hair to each peak and finding the position of the maximum for each incidence angle.



Calculate the displacement for each incidence angle by finding the difference between the zero degree position and the position of the maximum for each incidence angle. Once you have the displacements for each incidence angle next calculate the corresponding angle of refraction. The equation is provided in the **Discussion** section. Finally determine the index of refraction for each trial using **Snell's Law**. Also compute the mean and standard deviation of the index of refraction values. Be careful to not delete the graph or close the program until you have had your results checked.

Experiment O2a: Determining the Index of Refraction of Light

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheets: O2a: Determining the Index of Refraction of Light

NAME: ______________________________ DATE: __________________

Material: ________________________________

Thickness of Material, *t*: ________________________________

Theoretical Index of Refraction, μ: ________________________________

θ_i **angle of incidence**	**Position**	*d* **Displacement**	θ_r **angle of refraction**	**μ index of refraction**
0º		**X**	**X**	**X**
10 º				
15 º				
20 º				
25 º				
30 º				
35 º				
40 º				
45 º				
50 º				
55 º				

Mean for index of refraction (μ): ________________________________

Standard Deviation: ________________________________