O5a: Refraction of Light and its Application to Thin Converging Lenses

Introduction:

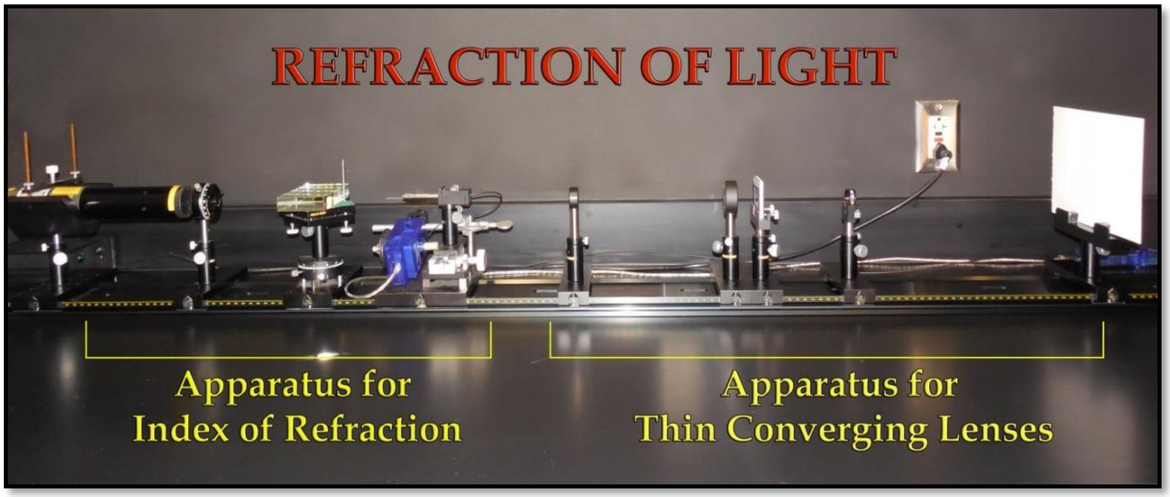
Light travels through the vacuum of empty space at the ultimate speed limit of the universe. The speed of light can change, though, depending upon the media light travels through. Light slows down differently each time it passes through other media (such as water or glass).

This difference in speed as light transitions from one medium to another causes a "bending" in a ray of light if the ray entering the different medium is at an angle other than 90°. This shifting, known as refraction of light, and the amount of angular deviation that the light ray refracts are both related to a dimensionless quantity called the index of refraction. This relationship is expressed via Snell's law, formulated by Dutch mathematician Willebrord Snellius in 1621 (though several others also discovered the law, including Rene Descartes in 1637).

The refraction of light at the boundary between two mediums can be used to control the path light takes. It is from this refraction of light that scientist have been able to construct optical instruments, which are designed to control light in very precise ways. Lenses, for example, are optical instruments that utilize a transparent medium, such as glass, in order to refract light in an exact way, so as to form an image. The shape of the lens and its refractive index govern the characteristics of the image that the lens will produce.

The purpose of this experiment is twofold. First, determine the index of refraction for a piece of clear glass. It will be found by measuring the displacement of a laser's light as it exits the glass, which is rotated to different angles of incidence. From the original angles of incidence θ_i and the distance the laser light is displaced, it is possible to determine the angles of refraction θ_r within the glass using geometry. With these two angles, then a simple process using Snell's Law will determine the index of refraction.

The focus of the second part of the experiment is to study the image characteristics produced by a thin converging lens, making use of an optical rail system of components. The data collected will allow the determination of the focal length for a thin converging lens and also the magnification of the image produced by that lens. The relationships between the object distance, image distance, and image size will then be examined graphically.



Picture 1

Apparatus:

- Laser & laser holder
- Two meter optical rail
- Multiple component carriers
- Mounting hardware
- Angular translator w/ prism mount
- Linear translator w/photometer mount
- Photometer sensor and rotational sensor
- Computer and interface
- Glass plate, polarizer, target & diffuser
- 1 10mm double convex (DCX) lens & lens holder
- 2 25mm double convex (DCX) lenses & lens holder

Discussion:

The index of refraction, η, is defined by the ratio of the light's speed in a vacuum to the light's speed in the medium. It is given by the following equation:

$$\eta = \frac{c}{v}$$

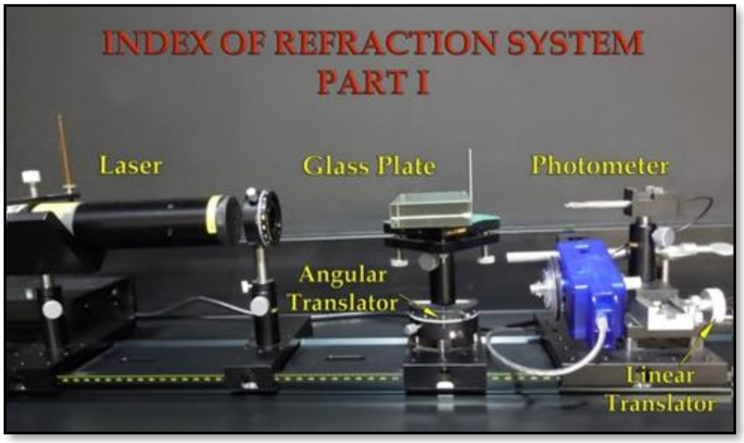
where: *c = speed of light in a vacuum*
v = speed of light in the medium

The bending of light, due to the speed change can be related to the angles, incidence and refraction, and the index in each medium, as described by Snell's Law:

Snell's Law of Refraction: $\eta_i \sin\theta_i = \eta_r \sin\theta_r$
(η is the index of refraction for a given medium)

Frequently the index of refraction's notation may be changed from η to μ indicating the index is relative to air: where $\eta_i \approx 1$ for air. In this case the index is calculated by the following:

$$\mu = \frac{\eta_r}{\eta_i} = \frac{\sin\theta_i}{\sin\theta_r}$$



Picture 2

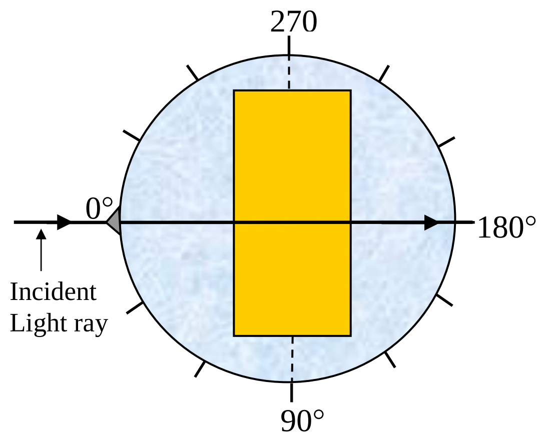


Figure 1

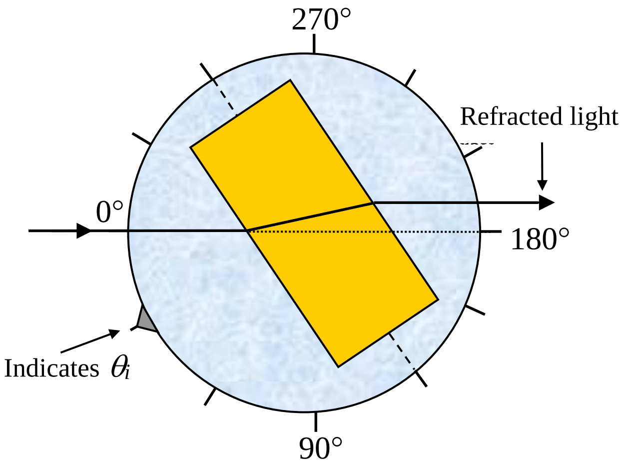


Figure 2

The material for Part I of this experiment is a thick glass plate approximately (6 x 11 x 2) centimeters. The glass will first be positioned on an angular translator, perpendicular (at 90°) to the incoming ray of light, where no refraction of the light beam occurs. Then the material will be rotated to several different angles (with the angle determined by the setting of the angular translator), causing light to be refracted within the material.

A linear translator, fitted with the eye of a photometer that can sense the intensity of the laser light, is set at the opposite end of the optical rail, and can carefully and slowly be moved in order to measure the position of the light ray as it changes for each of the different angles. The displacements of the ray of light, along with the thickness of the material, can then be used to calculate the angle of refraction, θ_r, for each incidence angle.

A top view representation of the light traveling through the glass while sitting on the angular translator has been provided in figures one and two.

From this representation in **Figure 2**, which has been expanded into **Figure 3**, in order to show it more clearly, it becomes possible to derive the equation necessary to find θ_r. Making use of the geometric fact that two intersecting lines form vertical angles which are congruent, it can be seen that for **Figure 3**: $\theta_i = \theta_r + \theta_\beta$. From this equation, it is not at all difficult to derive the following relationship that will be needed as θ_r is found:

$\theta_\beta = \theta_i - \theta_r$

Also apparent from closer consideration of **Figure 3** is the fact that the dotted line through the glass block is of length ***t***, where ***t*** represents the thickness of the block. Additionally, it can be seen that the shifting of the laser light has created two right triangles: the first has an angle of θ_r, an adjacent side with length ***t***, and a shared hypotenuse that has been labeled ***x***. The second triangle has an angle of θ_β, an opposite side of length ***d,*** where ***d*** is equal to the displacement of the laser light, and a shared hypotenuse ***x***.

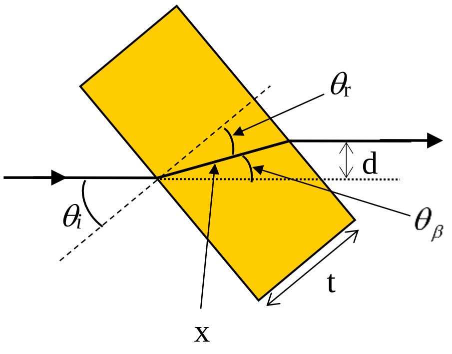


Figure 3

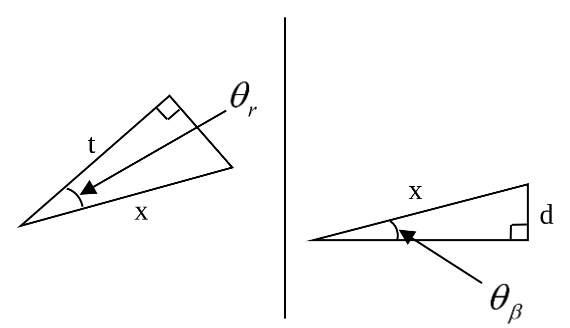


Figure 4

The layout of the two right triangles created by the laser light's displacement and shown in **Figure 3** can be seen much more clearly if one separates those two triangles out, as done in **Figure 4**.

From the provided picture and geometrical definitions, the following relationships may be stated:

$$\cos\theta_r = \frac{t}{x} \qquad \sin\theta_\beta = \frac{d}{x}$$

Now, by solving both equations so that x is isolated, and then setting the two equations equal to each other, the following result may be obtained:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin\theta_\beta}.$$

From here, one is able to make use of the fact that $\theta_\beta = \theta_i - \theta_r$, in order to find the following equation in terms of θ_i and θ_r:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin(\theta_i - \theta_r)}$$

At this point, it becomes necessary to consider the following trigonometric identity**:** $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$. By applying this identity, the above equation is transformed to yield up:

$$\frac{t}{\cos\theta_r} = \frac{d}{\sin\theta_i\cos\theta_r - \cos\theta_i\sin\theta_r}$$

Now, by cross multiplying, one can obtain that:

$$\frac{\sin\theta_i\cos\theta_r - \cos\theta_i\sin\theta_r}{\cos\theta_r} = \frac{d}{t}$$

And by then simplifying this equation, it becomes:

$$\sin\theta_i - \cos\theta_i\tan\theta_r = \frac{d}{t}$$

From here, one simply has to subtract the $\frac{d}{t}$ over to one side, add the $\cos\theta_i\tan\theta_r$ to the other side and divide out the $\cos\theta_i$ to isolate θ_r.

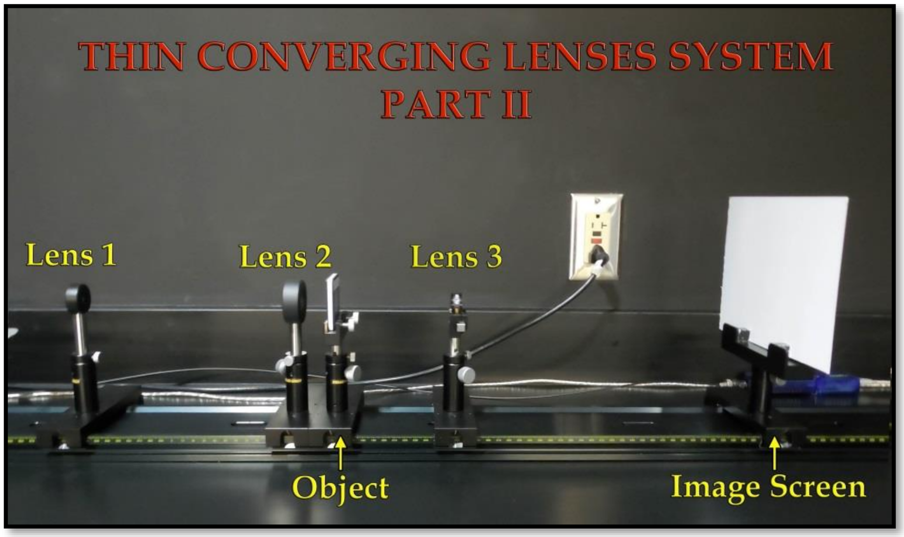
$$\frac{\sin\theta_i - \frac{d}{t}}{\cos\theta_i} = \tan\theta_r$$

This yields the needed equation to calculate the angle of refraction θ_r .

Angle of Refraction: $\theta_r = \tan^{-1}\left(\frac{\sin\theta_i - (d/t)}{\cos\theta_i}\right)$ Where:

d = displacement
t = thickness of material
θ_i = angle of incidence
θ_r = angle of refraction

After obtaining the angle of refraction use it together with the angle of incidence to calculate the index of refraction by using Snell's Law.



Picture 3

In Part II of the experiment the refraction of light is being used to control the light emanating from an object. The lens has been constructed in such a way that it will bend the light from the object to form a focused image, representing the original object, located at a specific position. There are two sequences of data collection for this part. The first sequence focuses on the distance and size of the image produced by a lens in relation to its position. The second sequence examines conjugate positions. Conjugate positions occur when the object position and image position remain fixed. With the image and object fixed and stationary, there are two lens positions that will produce a sharply focused image. These two lens positions are conjugate positions.

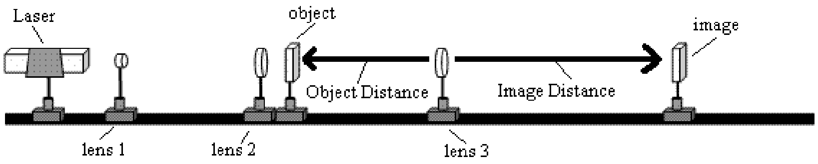


Figure 5

The optical rail system has many optical instruments mounted along its two meter length. Most of the instruments like lenses 1 & 2 will remain stationary throughout this part of the experiment. Lens 3 is the primary one being examined for this part and along with the image screen will be the only components moved. Using the diagram above one can see how to determine the object distance and image distance if the positions are known for the object, lens 3 and the image. During the sequence of trials these positions will be measure and subsequently the distances will be calculated.

The thin lens equation provides the relationship between the lens's focal length, which is a property of the lens, and the object distance and image distance.

Thin Lens Equation:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$$

where: f = *lens focal length*
d_o = *object distance*
d_i = *image distance*

The magnification factor for a lens or a lens system of multiple lenses is a ratio of the size of the image to the size of the original object. If the size of the object is known then it can be multiplied by the magnification factor to determine the size of the resulting image. The equation is provided following relationship:

Magnification Equation:

$$m=\frac{h_i}{h_o}=\frac{d_i}{d_o}$$

where: m = *magnification factor*
h_i = *image size*
h_o = *object size*
d_o = *object distance*
d_i = *image distance*

In order to help speed up the completion of this lab, a review of any physics textbook's discussion/summary of the chapter dealing with refraction, images, lens and other optical instruments is strongly suggested. For the Physics by Cutnell & Johnson see chapter 26.

Before beginning, please see the laboratory personnel for instructions concerning the proper handling and care of the optical equipment. Do not attempt to start this lab without checking with an instructor.

Procedures Part I:

1. Always wash hands before interacting with optics equipment. After having washed hands, **ask a lab instructor** for directions.
2. The thickness of the glass plate will be provided. **DO NOT ATTEMPT TO MEASURE THE THICKNESS OF THE GLASS**.
3. Position the angular translator to point at zero degrees.
4. Open the corresponding program for this experiment. This can be found within the experiments folder on the desktop.
5. Confirm the angular translator position is 0 degrees and adjust the linear translator position to be offset right of the laser beam ray. Start the computer data collection.
6. Begin collecting data by ***slowly and smoothly*** turning the knob of the sensor until you get a complete maximum peak (which will look like a tall hump) on the graph.
7. Now pause moving the linear translator but do not stop the computer collecting data.
8. Carefully position the angular translator to 10 degrees and repeat process (steps 6 & 7).
9. Continue each trial by increasing the incidence angle by 10 degrees and repeating the process (steps 6 & 7) until you finish with 50 degrees. You should have 6 peaks on the graph after completing the angles through 50 degrees.

Analysis Part I:

Once you have collected your data on the graph you need to calculate the light ray’s displacement from its original position due to refraction as it travels through the glass. The displacement is the difference in the position from the maximum peak at zero degrees to the maximum peak at each incidence angle.

First click on the “add a coordinates tool” ⌖ button. in the upper part of the graph. Then, from the drop-down list, select the “add multi-coordinates tool” option, as shown in **Figure 6**.

Position the selection box on the top of the first peak, corresponding to the zero degree maximum, **Figure 6**. This will be your zero degree position. The blue box underneath the horizontal axis will display the position relevant to that peak. Now move the selection box over to the next peak corresponding to the 10 degree maximum. Record this as the 10 degree position. Next move the selection box from the 10 degree peak to the 20 degree peak and record this next position. Continue these same steps, moving the selection box to each peak and finding the position of the maximum for each incidence angle.

Now, calculate the displacement for the 10 degrees incidence angle by finding the difference between the zero degree position and the position of the maximum at 10 degrees. Once you have the displacement, calculate the corresponding angle of refraction. The equation is provided in the **Discussion** section. Finally determine the index of refraction using **Snell’s Law**. **After completing the calculations for 10 degrees, verify these results with your lab instructor. The lab instructor will tell you how to proceed for the rest of the calculations!** Be careful not to delete the graph or close the program until you have had your results checked.

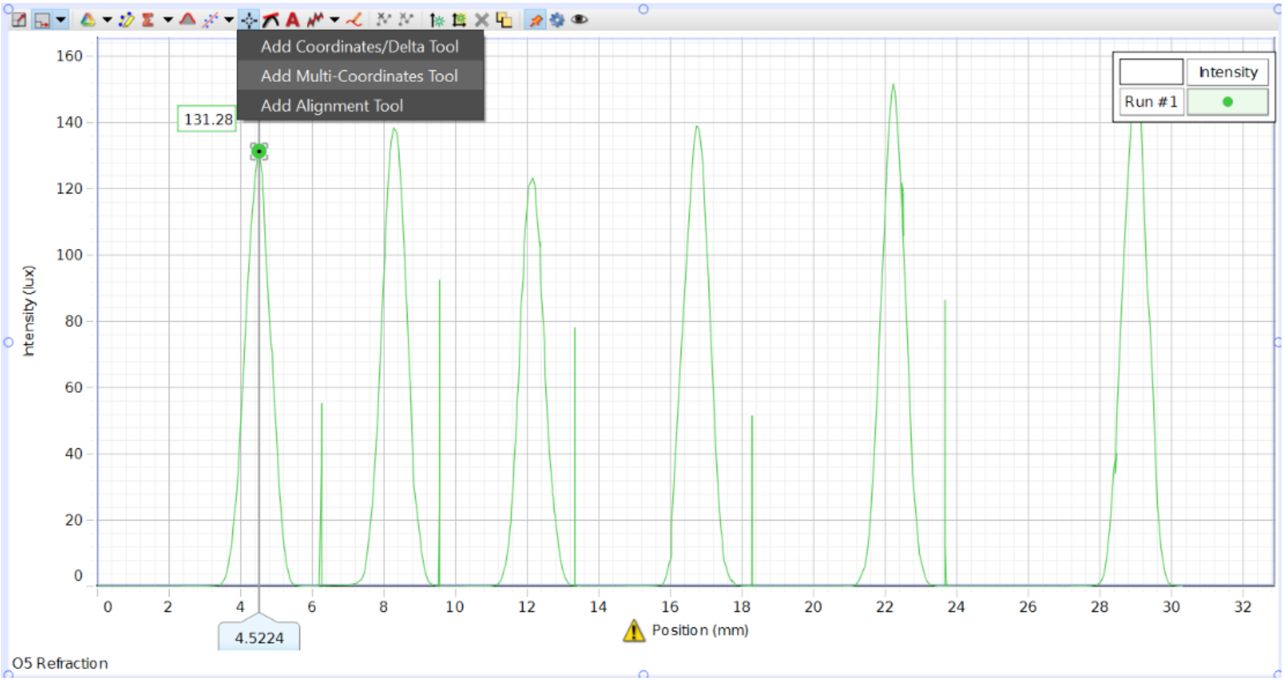

Add Coordinates/Delta Tool
Add Multi-Coordinates Tool
Add Alignment Tool
131.28
4.5224
Intensity (lux)
160
140
120
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
Position (mm)
Intensity
Run #1
05 Refraction

Figure 6

Experiment O5a: Index of Refraction of Light and its Application to Thin Converging Lenses

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor____

Experiment Start Date____________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheet: O5a-I: Index of Refraction of Light

NAME: ______________________________ **DATE:** __________________

Material: ________________________________

Thickness of Material t (mm)**:** ________________________________

Theoretical Index of Refraction η**:** ________________________________

angle of incidence θ_i	**Position** (mm)	**Displacement** d (mm)	**angle of refraction** θ_r	**index of refraction** η
0º		▨	▨	▨
▨	▨	▨	▨	▨
10 º				
20 º				
30 º				
40 º				
50 º				

Mean for index of refraction (η**):** ________________________________

Standard Deviation: ________________________________

Procedures Part II:

Verify the positions for the laser, lenses, object and other optical components are correct as specified on Table 1. Table 1 will be posted with the apparatus. It is not necessary to move or adjust any of the components other than Lens 3 and the image screen unless they are not at the specified positions. If any components are at different positions or misaligned please ask a lab assistant to help with the adjustments and confirm that the setup is correct. Part II of the experiment consists of two variations of steps: **Finding Image Positions** *&* **Finding Conjugate Positions**.

Finding Image Positions:

1. Move Lens 3 to the first position specified in Table 1.
2. Slowly move the image screen along the optical rail until the image of the target appears as sharply focused as possible.
3. Record the optical rail position of each optical component in the Data Table.
4. Use the caliper to measure the size of the target's circle outside diameter (OD) on the image screen. This will be an approximate measurement. Record this measurement as the image size in the Data Table.
5. Repeat these steps for each Lens 3 position listed in Table 1.

Finding Conjugate Positions:

1. Move the image screen to the position specified in Table 1. The object position will remain the same as before.
2. Slowly move Lens 3 along the optical rail until the image of the target appears as sharply focused as possible.
3. Record the optical rail position of each component and the approximate size of the image.
4. Without changing the position of the image screen, continue to slowly move Lens 3 further along the optical rail until the image again appears as sharply focused as possible, but at a different position from before.
5. Record all optical rail positions and the image size. The resulting image size for the second conjugate position will differ significantly from the image size of the first conjugate position.

Analysis Part II:

First, it is necessary to calculate the object distance and image distance for all positions. Calculate the object distance and the image distance by making use of the positions of the Object, the Image, and Lens 3 recorded in the Data Table, see **Figure 5**. The object distance and image distance should then be entered into the Analysis Table.

From these distances (object distance and image distance), it becomes possible to calculate the Focal length for Lens 3, by making use of the thin lens equation. Then calculate the magnitude of the magnification factor via the formula for magnification, using image distance and object distances. After calculating the magnification factor, calculate the theoretical image size by using the magnification factor and the object's original size.

Finally graph the image distance as a function of object distance and also graph the image size as a function of object distance. Use MS Excel to create these two graphs. Include the appropriate asymptotes on each graph.

Data Sheet: O5a-II: Thin Converging Lenses

NAME: ____________________________ **DATE:** _________________

Laser position: ____________________
Lens 1 position: ___________________ ***f*** = _____________
Lens 2 position: ___________________ ***f*** = _____________
Lens 3 focal length: - - - - - - - - - - - - > ***f*** = _____________
Object size: _______________________

Position (mm)			Size (mm)
Object	Lens 3	Image	Image (approx.)
Finding Image Positions:			
Finding Conjugate Positions:			

Analysis Sheet: O5a-II: Thin Converging Lenses

NAME: ______________________________ **DATE:** __________________

Distance (mm)		Focal length	Magnification Factor	Image Size (mm) (calculated)
Object	Image			
Finding Image Positions:				
Finding Conjugate Positions:				