S1a: Hooke's Law & Simple Harmonic Motion

Introduction:

This experiment examines Hooke's law and simple harmonic motion. Simple Harmonic Motion (SHM) is the motion when a position of a body can be described by a sinusoidal (or a cosinusoidal) function of time. This motion is over the same path, which includes an equilibrium position that the simple harmonic oscillator must pass through with each period of its motion. Examples of simple harmonic motion include the oscillation of a mass attached to an ideal spring (which will be studied in this experiment), the motion of a pendulum within a grandfather clock, the ticking of a metronome, and many others. SHM occurs whenever the net force or the net torque acting on a body is directly proportional to the displacement of the body from its equilibrium position. An example of such force, which will be studied in the lab, is the restoring force exerted by a stretched or a compressed spring, described by Hooke's law.

In this experiment, you will observe that the restoring force of the spring is indeed directly proportional to the amount of its stretch or compression, determine the spring constant, investigate how the period of oscillations depends on mass attached to the spring, observe oscillation patterns, and compare your experimental results with theoretical predictions.

Theoretical Background:

Hooke's law relates the force that a spring exerts on a body to the stretch of the spring. Robert Hooke (1635-1703), a contemporary of Isaac Newton, postulated a relationship between stress (the force applied to a spring) and strain (the stretching that results from the stress applied).

$$F_x = -kx \qquad (1).$$

In this equation F is the force exerted by the spring, $x=|L-L_0|$ is the absolute value of the difference between a length of a stretched or compressed spring, L, and its equilibrium length, L_0. The coefficient of proportionality, k, is called the spring constant, which is unique for each spring and describes how easy or hard to stretch or to compress the spring. Minus sign in the equation (1) reflects the fact that the restoring force tries to bring the spring back to its equilibrium position: compressed spring "pushes back" and stretched spring "pulls back".

Simple harmonic motion is characterized by a number of parameters. ***Amplitude***, *A*, is the maximum deviation of an object from its equilibrium position. ***Period***, *T*, is the time it takes to complete one full cycle of the motion. ***Frequency***, *f*, is the number of oscillations per second. ***Angular frequency***, ω, is related to the period as $\omega=2\pi/T$.

Equation, describing simple harmonic motion of a mass attached to an ideal spring, and the relationship between the period of oscillations and the mass attached to a light (of negligible mass) spring can be derived using Newton's Second Law. If the restoring force of the spring is the net force acting on a mass attached, then

$$F_{net} = -kx = ma \qquad (2).$$

Remembering that the acceleration is the second derivative of position of a particle, (2) can be re-written as

$$m\frac{d^2x(t)}{dt^2} + kx(t) = 0 \qquad (3).$$

Equation (3) is an example of a differential equation, which you will learn how to solve in math courses. One of solutions of equation (3) is $x(t)=A\cos(\omega t)$.Taking the second derivative and pugging expressions for x(t) and its second derivative in (3) one can see that this sets strict conditions on ω:

$$\omega = \sqrt{\frac{k}{m}} \qquad (4),$$

And, therefore, period of oscillation:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad (5).$$

If mass of the spring is not negligible, equation (5) needs to be adjusted by taking "effective mass", m_e, into account. In this experiment,

$$m_e = m_a + \frac{m_s}{3} \qquad (6),$$

where m_a is the mass attached and m_s is the mass of the spring.

If there is not any other forces present, the oscillations will continue forever. However, in real life, there is always some friction and air resistance present. This will cause oscillations to dampen: when the amplitude of oscillations will slowly decrease with time and oscillations will eventually stop.

Apparatus:

- Spring
- Mass hanger
- Masses (10 fifty-gram masses),
- Meter stick
- Support rods
- Computer timing system & photogate

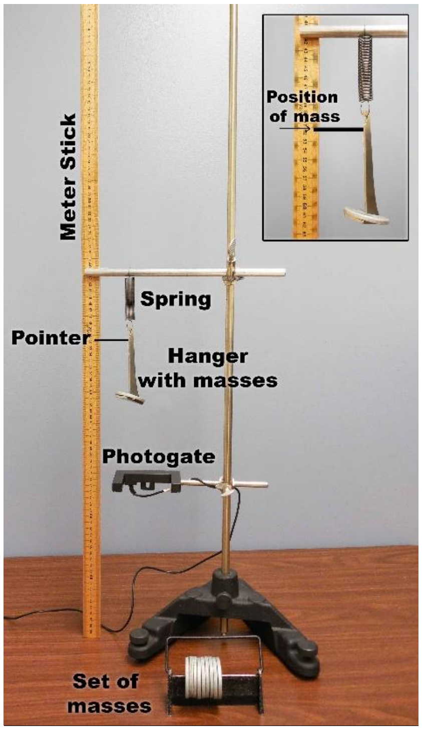

Figure 1

Procedures:

Part I: Hooke's Law

In the first part of the experiment, you will investigate Hooke's law by attaching different masses to a vertical spring and measuring the change in position of the pointer due to a stretch of the spring (see Figure 1). The horizontal cross bar should be placed approximately 60 centimeters above the tabletop. Adjust its height, if necessary.

1. Obtain the mass of the spring (the spring by itself) and record it on the **Data Sheet**.
2. Place the spring back on the cross bar.
3. Next, obtain and record the applied mass (including the mass of the mass hanger) for each of the ten trials. Start by adding a 50 gram mass to the mass hanger (trial #1), and then increase the applied mass by 50 grams for each trial. When recording these values, make sure to keep the masses in sequence (each mass is slightly different). DO NOT simply add the value written on the side of the masses to the previous total. This will give inaccurate results and require you to redo **Part I**. Instead, use a balance to find the mass of the hanger and the accumulated masses each time a mass is added.
4. Position the meter stick next to the spring, with the 0 centimeter mark up in the air and the 100 centimeter mark on the table. Place the mass hanger and the 1st mass on the spring, and record the position of the pointer (attached to the spring) relative to the meter stick.
5. Obtain the position for each of the ten trials. Use the masses that were measured before, keeping them in the same sequence as when they were measured.

Part II: Period of oscillations of a mass attached to a spring.

1. Start with the mass hanger and the first mass suspended on the spring, and move the photogate attached to the vertical bar such that when the spring is totally motionless, the pointer is interrupting the photogate (**only the pointer and not any other portion of the mass hanger**). The small diode light on the back of the photogate will stay lit when the alignment is correct.
2. Log on to the computer and open the appropriate data collection software program. Once the appropriate data collection program has been opened, make sure to open the activity connected with this experiment.
3. Carefully either pull down the mass hanger approx. one centimeter or raise the mass hanger approx. one centimeter, and then release it. Make sure to either pull or raise the mass hanger and spring straight up or down and not to either side. The spring should be bouncing *only* up and down. If there is any motion other than vertical motion, stop the spring and repeat. *One possible suggestion for creating only vertical oscillations: try using a pencil held horizontally to lift the spring one centimeter before releasing it.
4. Once stable oscillations have occurred, start the computer collection of data. Allow 50 to 100 data samples to be collected before stopping. Record oscillation data for each of the ten trials. The standard deviation for each trial should remain in the $10^{-3}\,\text{sec}.$ range or smaller for acceptable trials.

Analysis:

Part I: Hooke's Law

1. Using the data collected from **Part I** of this experiment, construct a graph (using *MS Excel*). This graph should have the applied mass on the y-axis and the corresponding position on the ***x***-axis.
2. Answer the following question in your analysis portion of the data sheets: What is the shape of your graph and is this shape in agreement of in disagreement with Hooke's Law? Explain.
3. Statistically analyze the graph in order to obtain the linear regression slope for the best fit line. Be sure to remember that all calculations should be done in the standard SI units. Also, it is important to note that the linear regression slope is used to calculate the spring constant; as can be seen on the attached **Data Sheet – Table 1**.
   To obtain the spring constant (k) from the linear regression slope, multiply the slope by the accepted value for gravity (9.792 m/s^2). The spring constant is a ***constant*** which has the same singular value for all of the different trials.

Part II: Period of oscillations of a mass attached to a spring.

1. For each mass attached, calculate the effective mass, $\boldsymbol{m_e}$, using equation (6). Record your data in the appropriate table.
2. For each trial, calculate the theoretical period of oscillation, using equation (5), your calculated $\boldsymbol{m_e}$ instead of $\boldsymbol{m}$, and the value of the spring constant $\boldsymbol{k}$ you determine in step (3). Compare your values for theoretical period with experimentally measured values. Reflect this comparison in your analysis page.
3. Construct a graph of the experimental period of oscillation as a function of effective mass, m_e. Use "power" function as a trendline to analyze your graph. Record the equation of the obtained fit.
4. Answer the following question (in the analysis portion). Is your obtained fit in agreement or disagreement with the theoretical prediction? Explain why.
5. Think about how you can determine the spring constant from your obtained fit and determine the spring constant. Show work! Compare your experimentally determined spring constant from the graph of period of oscillation as a function of the effective mass with that from the first graph (Part 1, step 3)). Calculate percent difference between the two values.

Experiment S1a: Hooke's Law & Simple Harmonic Motion

Student Name____

*Lab Partner Name*____

*Lab Partner Name*____

Physics Course____

Physics Professor______________________________________

Experiment Start Date__________________________________

Lab Assistant Name	*Date*	*Time In*	*Time Out*

Experiment Stamped Completed

Data Sheets: S1a: Hooke's Law & Simple Harmonic Motion

NAME: ____________________________ **DATE: ________________**

mass of the spring *(m_s)*: ____________________

Data Table 1.

applied mass (m_a) (Kg)	position of mass (x) (m)	experimental (mean) period of oscillation (s)	standard deviations (s)	# data points

Analysis:

I. Hooke's Law and Spring Constant

1. What is the shape of your graph and is this shape in agreement or in disagreement with Hooke's Law? Explain.

2. Linear regression:

   Slope of m_a vs x = __________________

   Spring constant:

   k = _______________

II. Oscillations

Analysis Table 1.

applied mass (m_a) ( )	effective mass (m_e) ( )	experimental period of oscillation (copy from Data table 1) ( )	Theoretical period of oscillations ( )	% error (theoretical and experimental period)

1. Obtained fit (using power trendline):

2. Is your obtained fit in agreement or disagreement with the theoretical prediction? Explain why.

3. Calculations of the spring constant from the obtained fit:

4. Calculations of % difference between spring constants obtained from two graphs: